A BUCKLING LOAD FORMULA FOR SIMPLY SUPPORTED PLATES UNDER COMBINED AXIAL LOAD AND NORMAL PRESSURE

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ABSTRACT

This is a theoretical study to determine a practical design formula for the buckling load of simply supported rectangular plates under combined axial load and normal pressure.

The theory and method used was based on Samuel Levy's integrated solution of the non-linear differential equations as derived by von Karman. The length-to-width ratio (a/b) of the plates considered was extended up to a value of 4:1. For each length-to-width ratio a set range of normal pressures was used in obtaining the different buckling loads.

The study confirmed the conclusion arrived at by Levy; that is, the normal pressure increases the buckling load. It was further observed and concluded that for low values of a/b (less than 1.0) the buckling loads obtained in the manner described in Chapter III are impractical to attain. Yield failure will probably occur before the calculated critical load is reached.

In general, however, the authors' recommended buckling load formula for a steel plate is of the form,

$$\sigma_c = \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{h}{b}\right)^2 K$$

where the value of K is scaled from Fig. XXXV. The basic limitation on the accuracy of this value of K is the approximated expression(s) of the deflection equation. The computed value of σ_c must be compared with the yield strength of the material since the validity of the theory only holds true within the elastic region.

Thesis Supervisor: J. Harvey Evans
Title: Professor of Naval Architecture

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I. INTRODUCTION

Background

The discussion of buckling of plates is usually based upon a linear differential equation derived under the assumption that the deflections of the plates are small in comparison with their thickness. In particular, present design practice on hull plating under combined normal pressure and axial load is often based on approximate critical buckling load formulas. The normal load has generally been relegated to minor roles. The reasons purported to supported this were very well founded, both from the standpoints of theory and practice. Samuel Levy and others [2], [3] arrived at the conclusion that normal pressure increases the buckling load and thereby indicated that it would be conservative design to neglect the effect of normal pressure. The same conclusion was stated by Bleich [1]. Bleich further made mention of the fact that deflection of ship's hull plating usually does not exceed one-half the plate's thickness and therefore concluded that linearized theory could be applied.

It is worthy to note at this point, however, that deflections exceeding half the plate's thickness are possible, especially in the case of simply supported plates. In those cases linear theory of plates no longer applies. As will be briefly explained in the subsequent paragraph, the problem becomes a non-linear stress problem since deflections of the order of magnitude of the plate thickness must be considered.

Briefly stated, the nonlinearity of the differential equations has its origin in the fact that, in the case of large deflections, there is an interaction between the membrane stresses and the curvature of the plate. This interaction leads to non-linear terms in the equations of

equilibrium of the plate elements.

The complete differential equations of the problem were formulated by von Karman, who added those non-linear terms pertaining to the flexural rigidity of the plate. Timoshenko [6], Marguerre and Trefftz [8] derived the expressions for the strain energy of plates with large deflections.

An attempt to develop the large deflection theory for rectangular plates under combined bending and longitudinal compression was made by Bengston [7]. His studies included plates with simply supported and clamped edges. However, Bengston introduced, in the course of his analysis, certain arbitrary assumptions which in part contradict each other and it is very doubtful whether the results of his analysis can be considered as entirely correct.

In a series of papers Samuel Levy gave exact theoretical solutions for rectangular plates with large deflections. The theories developed include simply supported and clamped plates.

The problem of rectangular plates carrying longitudinal compression and normal pressure is of prime importance in the design of the hull plating of ships. It is closely related to the question of buckling strength of plates.

Statement of the Problem

It is common knowledge that factors of safety are actually factors of ignorance which in some cases have remained unchanged for a period of time for no obvious reason. Factors of safety may be used to account for one or more of the following: (a) material imperfections, (b) faulty construction practices, (c) material degradations due to corrosion, and (d) lack of ability to make rigorous calculations. The conservativeness of design as previously mentioned prompted the authors to ask in what way would conservative design affect the factor of safety; that is, does conservative design actually compound the factor of safety? If, for instance the safety factor is increased by only a few tenths of a percent, then one is justified in using linear theory. However, if the increase becomes considerable, the design may become uneconomical.

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With the salient points stated in the previous paragraph in mind and also with the conclusion of Levy on the effect of normal pressure on the buckling load of plates, the authors came to the conclusion that there is a need for further investigations. First, these investigations afford the possibility of checking the accuracy of simpler approximate methods and formulas. Secondly, and actually the ultimate goal of this thesis, these investigations would hopefully lead to a formulation of a simple design formula for plates subjected to the simultaneous action of normal and edge loads.

A literature survey disclosed there is no simple formula which a designer can use with great facility. Fortunately, general solutions to the differential equations are available in the form of Fourier series. Though the analysis of these solutions is highly involved and the numerical work for obtaining special solutions is very laborious, a method (Chapter III) has been devised which lends itself to computer programming.

This thesis is concerned mainly with plates simply supported on all edges and subjected to combined loadings of uniform normal pressure and axial load in one direction. It is further hoped that it will serve as a forerunner of later investigations based on different boundary conditions.

II. THEORY

Nomenclature (See Fig. I):

a	plate length in the x-direction.
b	plate width in the y-direction.
α =	a/b
h	plate thickness.
w	plate deflection.
x, y	coordinate axes with origin at corner of plate.
E	Young's modulus.
μ	Poisson's ratio. (Note: Tabulated results and figures are for steel with μ = 0.300)
p	uniform normal pressure on plate.
e	average compressive strain at edges y = 0, b.
P	axial load on plate.
F	stress function.
D =	$Eh^3/12\ (1 - \mu^2)$, Flexural rigidity of the plate.

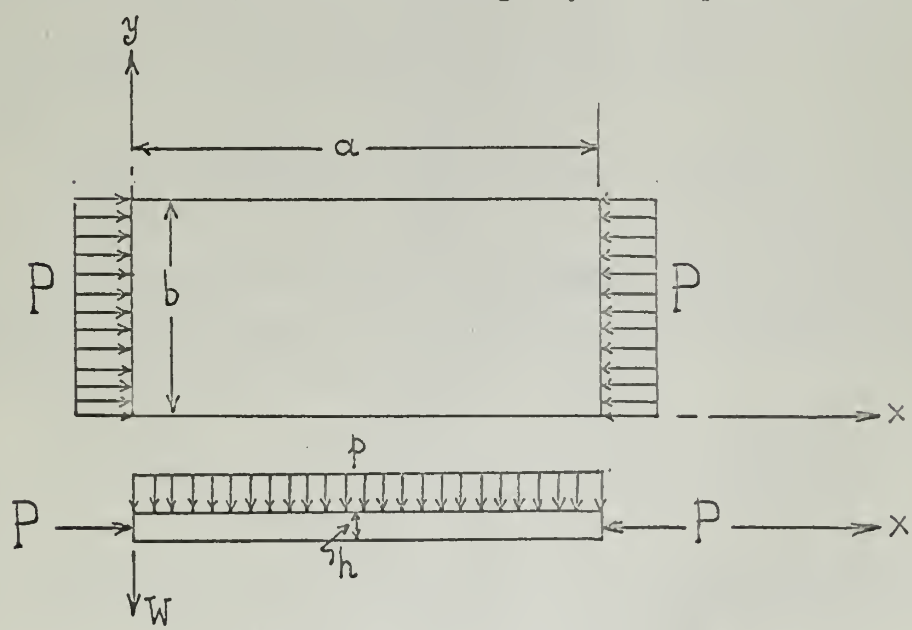

Fig. I Schematic diagram of plate under axial load and normal pressure.

Fundamental Equations

The fundamental partial differential equations governing the deformations of flat plates have been derived by von Karman. They are given by Samuel Levy [2] and Timoshenko [6]. The exact mathematical analysis to determine the buckling strength of a simply supported flat plate under combined edge compression and normal loading involves the integration of these equations; viz.,

$$\frac{\partial^4 F}{\partial x^4} + 2\frac{\partial^4 F}{\partial x^2 \partial y^2} + \frac{\partial^4 F}{\partial y^4} = E\left[\left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right] \tag{1}$$

$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{p}{D} + \frac{h}{D}\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right) \tag{2}$$

The boundary conditions for a simply supported rectangular plate to be satisfied by equations (1) and (2) are for deflection, w, and edge bending moments per unit length to be zero at the edges of the plate; viz.,

$$m_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \mu\frac{\partial^2 w}{\partial y^2}\right) = 0 \quad \text{at } x = 0,\ x = a \tag{3}$$

$$m_y^y = -D\left(\frac{\partial^2 w}{\partial y^2} + \mu\frac{\partial^2 w}{\partial x^2}\right) = 0 \quad \text{at } y = 0,\ y = b \tag{4}$$

The complete solution is more fully explained in reference [2] and, without loss of generality, the deflection equation is here only reproduced in the following Fourier series,

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} w_{m,n} \sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{5}$$

where the undetermined constants, $w_{m,n}$, must satisfy the relation(s) expressed by equation (9) of the same reference.

The average compressive stain, e, at the edges y = 0, b is computed from equation (11) of [2] as:

$$e = \frac{P}{Ebh} + \frac{\pi^2}{8\alpha^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m^2 w_{m,n}^2 \tag{6}$$

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The fundamental partial differential equations governing the deformations of flat plates have been derived by von Karman. They are given by Samuel Levy [2] and Timoshenko [6]. The exact mathematical analysis to determine the buckling strength of a simply supported flat plate under combined edge compression and normal loading involves the integration of these equations; viz.,

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$$\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D} + \frac{h}{D}\left(\frac{\partial^2 F}{\partial y^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 w}{\partial x \partial y}\right) \quad (2)$$

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$$e = \frac{P}{Ebh} + \frac{\pi^2}{8a^2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} m^2 w_{m,n}^2 \quad (6)$$

In this thesis, the deflection equation for large values of a/b ($\alpha >> 1$) is approximated by the following expression,

$$w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b}$$

$$+ w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + w_{2,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b} , \qquad (7)$$

and for a/b - values in the vicinity of 1 and less than 1, the approximated deflection equation is,

$$w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{1,3} \sin \frac{\pi x}{a} \sin \frac{3\pi y}{b}$$

$$+ w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b} + w_{3,3} \sin \frac{3\pi x}{a} \sin \frac{3\pi y}{b}$$

$$+ w_{1,5} \sin \frac{\pi x}{a} \sin \frac{5\pi y}{b} + w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} . \qquad (8)$$

There are no special reasons for using two deflection equations for different ranges of α values, except that equation (7) restricts the deflected shape of the plate to one sine wave across the width and a combination of four sine waves along its length. The errors involved by this approximation are expected to be less than five percent [2], [3]. For this reason, equation (7) could be reduced to a fewer number of terms as α is reduced. This is evidenced by the deflection equation used for $\alpha = 3.0$ in reference [2]. More specifically, the contribution of the higher-ordered deflection coefficients in the x-direction becomes less significant as α is reduced from an $\alpha >> 1$.

However, as α is further reduced the assumption of one sine wave across the width of the plate becomes incorrect since the higher-ordered deflection coefficients in the y-direction become increasingly significant and cannot be neglected unless the desired degree of accuracy is sacrificed. Thus, for such α values, equation (8) is used. As before, it is believed that the errors incurred by using a finite number of deflection coefficients would give results within the desired accuracy (see Table 13 of [2]).

The determination of the value of α applicable to equation (7) or (8) and the more general case of expanding the work of Levy et al on simply supported rectangular plates within the practical values of a/b are problems examined in this thesis.

III. PROCEDURE

With some additional steps and explanatory comments incorporated, the procedure followed in solving for the deflection equations was essentially the same as that outlined in reference [3]. The steps were as follows:

1. The family of four or six simultaneous cubic equations corresponding to the same number of unknown deflection coefficients was first obtained. This was done by solving for the coefficients $b_{p,q}$, defined by equation (8) of reference [2], and substituting them into equation (9) of the same reference. The results are shown in Appendix A.

2. Each of the resulting equations was divided by h^3. This step nondimensionalized the family of equations.

3. Values of $w_{1,1}/h$, $w_{1,3}/h$, $w_{3,1}/h$, etc. were estimated corresponding to chosen values of Pb/Eh^3 and pb^4/Eh^4 -- the non-dimensional axial load and normal pressure, respectively. This was the most delicate step since at higher values of Pb/Eh^3 (normal pressure held constant) more than one solution was found possible (see [3] also). In other words, the estimated w's, when not properly chosen, could lead to solutions other than those corresponding to a continuous change in buckled form from zero axial load to the buckling load.

It was therefore decided that a step-by-step method of estimating the w's be employed. In conjunction with the subsequent steps 4, 5, and 6, this method proceeds as follows:

a). Corresponding to three low values of Pb/Eh^3, including $Pb/Eh^3 = 0$, three sets of solutions were obtained using a reasonable set of w-estimates. These values of Pb/Eh^3 were hoped to lead to unique solutions. For these estimated w's, reference [2] or [3] was used as a guide.

b). From (a) above, succeeding estimates of w's as axial load is increased, were based on the continued trend of each w-vs-Pb/Eh^3 curve; viz., the slope and the rate of change of slope computed from the last three correct points were employed in an abridged Taylor series of the form,

$$w(x_0 + h) = w(x_0) + w'(x_0)\frac{h}{1!} + w''(x_0)\frac{h^2}{2!},$$

where, $w(x_0 + h)$ = next estimate of the particular w.

$w(x_0)$ = last computed value of w.

$w'(x_0)$ = slope obtained from the last two computed points.

$w''(x_0)$ = rate of change of slope based on the last three computed values of w.

4. The resulting cubic equations shown in Appendix A were linearized by expanding the right-hand side of each equation in a Taylor series in the neighborhood of the estimated values of $w_{1,1}/h$, $w_{1,3}/h$, $w_{3,1}/h$, etc., omitting higher ordered terms.

5. Crout's method [5] was used in solving for the difference between the estimated w's and their improved values.

6. Step 5 was repeated until such time that the calculated error sum was less than a test constant. The error sum was defined by the authors as the measure of closeness of successive approximations. In equation form, it is as follows:

$$\text{Error} = \sum_{i=1}^{N} x_i$$

where, x_i represents the absolute value of the difference between an estimate w and its improved value.

The test constant used for equation (7) was 0.0001 and for equation (8), 0.000001. Why two different values of the test constant

were used would become evident from a study of the results in Chapter IV.

7. The average strain corresponding to each axial load was computed from equation (6).

8. In the vicinity of the buckling load where the strain started to change more rapidly than the axial load, the $w_{1,1}/h$ was made the independent variable in place of Pb/Eh^3. The other deflection coefficients were computed in a similar fashion as before.

Values of the buckling load were obtained for values of pb^4/Eh^4 equal to 2.50, 7.50, 12.50, 18.00, 24.50, and 30.00, and at values of a/b up to 4.00. These were plotted versus a/b and analyzed for a possible comparison with Bryan's classical solution.

The whole procedure(s) had been programmed with the use of the IBM-7094. Details not otherwise covered here may be found in the programs shown in Appendix B.

IV. RESULTS

Tables I to XII are sample results of the computer programs. Tables I to VI were obtained using equation (8) for an $a/b = 1.00$ and, the rest of the tables were for an $a/b = 4.00$ using equation (7).

Figures II to XXXIII are the graphical results of the same programs. The curves drawn are the stress-strain curve analogy for plates with the non-dimensional axial load, Pb/Eh^3, as ordinate and the non-dimensional average strain, eb^2/h^2, as abscissa. (Note: Not all results for the chosen a/b-values used were plotted.)

Tabulated values of the critical loads, rounded to the nearest hundredth, are shown in Tables XII and XIV.

Figure XXXIV is a plot of the critical Pb/Eh^3 versus a/b. The curves between zero a/b and the smallest value of a/b used have been extrapolated. The curves have been faired at points where the results of both equations (7) and (8) either become tangent or intersect. Refinements of the curves in the vicinity of the cusps were made by obtaining more results at the a/b-values concerned. These results are shown in Tables XIII and XIV.

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Figure XXXIV is a plot of the critical $P_c b/Eh^3$ versus a/b. The curves between zero a/b and the smallest value of a/b used have been extrapolated. The curves have been faired at points where the results of both equations (7) and (6) either become tangent or intersect. Refinements of the curves in the vicinity of the cusps were made by obtaining more results at the a/b-values concerned. These results are shown in Tables XIII and XIV.

Table I - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 1.00$, $\mu = 0.300$. Normal pressure, $p = 2.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.113109	0.001522	0.001522	0.000156	0.000134	0.000134	-0.015
0.500	0.130955	0.001535	0.001606	0.000159	0.000135	0.000137	0.521
1.000	0.155300	0.001552	0.001704	0.000162	0.000135	0.000140	1.029
1.500	0.190218	0.001580	0.001826	0.000166	0.000135	0.000143	1.544
2.000	0.243439	0.001636	0.001995	0.000170	0.000135	0.000146	2.073
2.500	0.329375	0.001781	0.002298	0.000177	0.000135	0.000150	2.633
3.000	0.466786	0.002213	0.003028	0.000195	0.000136	0.000154	3.268
3.500	0.651813	0.003327	0.004820	0.000246	0.000137	0.000161	4.024
4.000	0.848735	0.005355	0.008196	0.000384	0.000143	0.000176	4.889
4.500	1.034310	0.008183	0.013194	0.000667	0.000154	0.000205	5.821
5.000	1.204318	0.011610	0.019678	0.001151	0.000172	0.000252	6.793
5.500	1.360460	0.015481	0.027524	0.001886	0.000196	0.000320	7.792
6.000	1.505171	0.019690	0.036648	0.002916	0.000227	0.000412	8.810
6.500	1.640526	0.024166	0.046998	0.004283	0.000264	0.000530	9.845
7.000	1.768140	0.028864	0.058539	0.006030	0.000308	0.000674	10.896
7.500	1.889255	0.033752	0.071252	0.008198	0.000359	0.000846	11.961
8.000	2.004827	0.038810	0.085125	0.010828	0.000420	0.001046	13.042
8.500	2.115604	0.044023	0.100152	0.013961	0.000492	0.001273	14.137
9.000	2.222177	0.049379	0.116331	0.017636	0.000580	0.001528	15.248
9.500	2.325014	0.054867	0.133658	0.021890	0.000688	0.001807	16.376
10.000	2.424491	0.060477	0.152130	0.026757	0.000821	0.002110	17.521
10.500	2.520910	0.066194	0.171741	0.032266	0.000986	0.002434	18.684
11.000	2.614510	0.071999	0.192479	0.038442	0.001190	0.002775	19.867
11.500	2.705480	0.077869	0.214327	0.045301	0.001442	0.003128	21.070
12.000	2.793967	0.083776	0.237259	0.052850	0.001751	0.003489	22.295
12.500	2.880085	0.089685	0.261241	0.061089	0.002128	0.003851	23.543
13.000	2.963914	0.095555	0.286231	0.070006	0.002583	0.004206	24.813
13.500	3.045514	0.101342	0.312181	0.079581	0.003127	0.004546	26.108
14.000	3.124924	0.106995	0.339036	0.089786	0.003771	0.004861	27.427
14.500	3.202167	0.112462	0.366738	0.100586	0.004525	0.005139	28.772
15.000	3.277257	0.117688	0.395229	0.111941	0.005399	0.005370	30.142
15.500	3.350196	0.122617	0.424455	0.123810	0.006404	0.005540	31.537
16.000	3.420982	0.127192	0.454368	0.136152	0.007549	0.005637	32.957

Table I (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 1.00$, $\mu = 0.300$. Normal pressure, $p = 2.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
16.500	3.489605	0.131359	0.484926	0.148926	0.008843	0.005646	34.402
17.000	3.556047	0.135060	0.516103	0.162099	0.010296	0.005552	35.873
17.500	3.620286	0.138240	0.547881	0.175642	0.011916	0.005341	37.369
18.000	3.682288	0.140841	0.580262	0.189534	0.013714	0.004996	38.891
18.500	3.742007	0.142805	0.613260	0.203761	0.015701	0.004502	40.438
19.000	3.799381	0.144069	0.646908	0.218318	0.017886	0.003841	42.011
19.500	3.854328	0.144565	0.681258	0.233211	0.020285	0.002996	43.611
20.000	3.906737	0.144218	0.716381	0.248451	0.022910	0.001948	45.239
20.500	3.956468	0.142943	0.752370	0.264062	0.025780	0.000677	46.897
21.000	4.003334	0.140647	0.789343	0.280075	0.028913	-0.000837	48.586
21.500	4.047101	0.137221	0.827445	0.296530	0.032333	-0.002613	50.310
22.000	4.087464	0.132543	0.866846	0.313473	0.036063	-0.004672	52.070
22.500	4.124046	0.126482	0.907749	0.330955	0.040133	-0.007032	53.871
23.000	4.156369	0.118897	0.950386	0.349025	0.044572	-0.009707	55.717
23.500	4.183853	0.109654	0.995010	0.367720	0.049409	-0.012703	57.612
24.000	4.205803	0.098647	1.041883	0.387049	0.054667	-0.016014	59.562
24.500	4.221416	0.085840	1.091255	0.406970	0.060358	-0.019621	61.571
25.000	4.229790	0.071321	1.143337	0.427361	0.066472	-0.023485	63.643
25.500	4.229896	0.055357	1.198303	0.447995	0.072967	-0.027555	65.779
26.015	4.219896	0.037904	1.258260	0.469164	0.079986	-0.031938	68.049
26.282	4.209896	0.028733	1.290778	0.479911	0.083709	-0.034291	69.250
26.479	4.199896	0.021994	1.315644	0.487706	0.085491	-0.036091	70.151
26.638	4.189895	0.016648	1.336290	0.493858	0.088743	-0.037593	70.885
26.771	4.179895	0.012261	1.354111	0.498900	0.090636	-0.038904	71.506
26.885	4.169895	0.008598	1.369846	0.503121	0.092259	-0.040080	72.043
26.984	4.159895	0.005512	1.383944	0.506695	0.093668	-0.041154	72.513
27.070	4.149895	0.002903	1.396704	0.509740	0.094900	-0.042148	72.927
27.145	4.139895	0.000699	1.408341	0.512342	0.095983	-0.043078	73.295
27.211	4.129895	-0.001158	1.419012	0.514563	0.096937	-0.043956	73.622
27.270	4.119895	-0.002711	1.428842	0.516454	0.097777	-0.044790	73.914
27.320	4.109895	-0.003996	1.437929	0.518054	0.098517	-0.045587	74.173
27.365	4.099895	-0.005043	1.446354	0.519395	0.099167	-0.046352	74.404
27.403	4.089895	-0.005877	1.454183	0.520505	0.099737	-0.047089	74.609

Table I (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 1.00$, $\mu = 0.300$. Normal pressure, $p = 2.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
27.437	4.079895	-0.006518	1.461472	0.521407	0.100233	-0.047802	74.790
27.465	4.069895	-0.006986	1.468271	0.522120	0.100663	-0.048493	74.949
27.489	4.059895	-0.007297	1.474621	0.522662	0.101031	-0.049164	75.088
27.509	4.049895	-0.007465	1.480560	0.523047	0.101344	-0.049818	75.209
27.524	4.039895	-0.007503	1.486119	0.523290	0.101605	-0.050457	75.313
27.537	4.029895	-0.007422	1.491328	0.523401	0.101819	-0.051080	75.402
27.546	4.019895	-0.007231	1.496211	0.523390	0.101989	-0.051691	75.475
27.551	4.009895	-0.006940	1.500794	0.523268	0.102118	-0.052288	75.535
27.554	3.999895	-0.006557	1.505095	0.523043	0.102210	-0.052874	75.582
27.555	3.989895	-0.006090	1.509135	0.522723	0.102266	-0.053449	75.617
27.552	3.979894	-0.005545	1.512931	0.522314	0.102291	-0.054014	75.641

Table II - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 1.00$, $\mu = 0.300$. Normal pressure, $p = 7.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{e b^2}{h^2}$
0.	0.329069	0.004721	0.004721	0.000485	0.000403	0.000403	0.133
0.500	0.375274	0.004842	0.005064	0.000499	0.000404	0.000412	0.674
1.000	0.433827	0.005031	0.005516	0.000518	0.000404	0.000421	1.232
1.500	0.508237	0.005343	0.006154	0.000545	0.000405	0.000430	1.819
2.000	0.601481	0.005870	0.007116	0.000587	0.000406	0.000441	2.446
2.500	0.714013	0.006741	0.008610	0.000658	0.000408	0.000454	3.129
3.000	0.842155	0.008089	0.010891	0.000781	0.000411	0.000472	3.876
3.500	0.979252	0.009994	0.014177	0.000988	0.000418	0.000496	4.685
4.000	1.118822	0.012457	0.018599	0.001321	0.000429	0.000531	5.548
4.500	1.256523	0.015424	0.024205	0.001821	0.000446	0.000580	6.454
5.000	1.390149	0.018820	0.030995	0.002531	0.000467	0.000646	7.395
5.500	1.518869	0.022574	0.038954	0.003492	0.000494	0.000731	8.363
6.000	1.642574	0.026629	0.048062	0.004746	0.000527	0.000839	9.355
6.500	1.761487	0.030937	0.058305	0.006331	0.000566	0.000970	10.367
7.000	1.875953	0.035466	0.069672	0.008287	0.000612	0.001126	11.397
7.500	1.986346	0.040189	0.082167	0.010655	0.000667	0.001308	12.445
8.000	2.093025	0.045088	0.095759	0.013473	0.000732	0.001515	13.510
8.500	2.196316	0.050148	0.110476	0.016780	0.000809	0.001749	14.593
9.000	2.296504	0.055356	0.126310	0.020613	0.000904	0.002007	15.692
9.500	2.393836	0.060701	0.143262	0.025007	0.001019	0.002290	16.809
10.000	2.488521	0.066170	0.161328	0.029994	0.001160	0.002594	17.944
10.500	2.580733	0.071745	0.180504	0.035602	0.001335	0.002919	19.099
11.000	2.670615	0.077408	0.200779	0.041851	0.001550	0.003260	20.273
11.500	2.758281	0.083135	0.222136	0.048757	0.001813	0.003613	21.469
12.000	2.843819	0.088897	0.244550	0.056327	0.002133	0.003973	22.686
12.500	2.927293	0.094660	0.267990	0.064559	0.002520	0.004334	23.927
13.000	3.008751	0.100383	0.292416	0.073442	0.002985	0.004688	25.190
13.500	3.088220	0.106024	0.317781	0.082957	0.003536	0.005028	26.478
14.000	3.165716	0.111533	0.344034	0.093078	0.004186	0.005344	27.790
14.500	3.241245	0.116862	0.371120	0.103773	0.004942	0.005625	29.127
15.000	3.314803	0.121956	0.398934	0.115003	0.005816	0.005860	30.489
15.500	3.386382	0.126763	0.427576	0.126731	0.006817	0.006037	31.876
16.000	3.455968	0.131229	0.456848	0.138918	0.007954	0.006143	33.289

Table II (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, a/b = 1.00, μ = 0.300. Normal pressure, $p = 7.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
16.500	3.523541	0.135299	0.486762	0.151527	0.009236	0.006165	34.726
17.000	3.589080	0.138921	0.517291	0.164526	0.010672	0.006088	36.188
17.500	3.652556	0.142039	0.548417	0.177889	0.012271	0.005897	37.676
18.000	3.713931	0.144599	0.580139	0.191594	0.014042	0.005579	39.188
18.500	3.773159	0.146544	0.612469	0.205629	0.015995	0.005116	40.726
19.000	3.830180	0.147813	0.645434	0.219990	0.018142	0.004493	42.289
19.500	3.884915	0.148342	0.679081	0.234680	0.020494	0.003692	43.879
20.000	3.937264	0.148057	0.713474	0.249711	0.023066	0.002695	45.497
20.500	3.987097	0.146879	0.748695	0.265104	0.025874	0.001484	47.143
21.000	4.034348	0.144717	0.784851	0.280889	0.028936	0.000040	48.821
21.500	4.078508	0.141468	0.822070	0.297103	0.032274	-0.001658	50.531
22.000	4.119609	0.137016	0.860509	0.313792	0.035911	-0.003627	52.277
22.500	4.157217	0.131232	0.900349	0.331006	0.039873	-0.005885	54.062
23.000	4.190920	0.123980	0.941796	0.348794	0.044191	-0.008447	55.891
23.500	4.220217	0.115123	0.985077	0.367198	0.048890	-0.011318	57.767
24.000	4.244515	0.104545	1.030425	0.386239	0.053997	-0.014494	59.695
24.500	4.263144	0.092182	1.078057	0.405896	0.059526	-0.017955	61.680
25.000	4.275380	0.078069	1.128141	0.426081	0.065474	-0.021661	63.725
25.500	4.280478	0.062397	1.180770	0.446614	0.071809	-0.025553	65.831
25.550	4.280566	0.060758	1.186174	0.448676	0.072461	-0.025950	66.045
25.600	4.280573	0.059106	1.191604	0.450739	0.073117	-0.026348	66.259
26.343	4.270575	0.033610	1.275424	0.481195	0.083174	-0.032375	69.518
26.641	4.260573	0.023209	1.310761	0.493098	0.087319	-0.034850	70.860
26.859	4.250573	0.015668	1.337450	0.501628	0.090380	-0.036706	71.856
27.035	4.240574	0.009719	1.359503	0.508332	0.092848	-0.038241	72.667
27.183	4.230573	0.004853	1.378486	0.513818	0.094917	-0.039573	73.353
27.309	4.220573	0.000801	1.395211	0.518406	0.096689	-0.040763	73.946
27.419	4.210573	-0.002606	1.410168	0.522289	0.098225	-0.041847	74.465
27.514	4.200573	-0.005481	1.423684	0.525598	0.099567	-0.042849	74.924
27.599	4.190573	-0.007906	1.435988	0.528424	0.100745	-0.043785	75.332
27.673	4.180573	-0.009946	1.447253	0.530839	0.101782	-0.044669	75.694
27.738	4.170573	-0.011650	1.457613	0.532896	0.102695	-0.045509	76.018
27.796	4.160573	-0.013058	1.467173	0.534639	0.103498	-0.046313	76.303

Table II (cont.) - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 1.00$, $\mu = 0.300$. Normal pressure, $p = 7.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
27.846	4.150573	-0.014204	1.476021	0.536103	0.104203	-0.047085	76.563
27.890	4.140573	-0.151116	1.484228	0.537317	0.104821	-0.047830	76.791
27.928	4.130573	-0.015817	1.491858	0.538308	0.105358	-0.048551	76.993
27.960	4.120573	-0.016330	1.498961	0.539097	0.105823	-0.049252	77.171
27.988	4.110573	-0.016670	1.505584	0.539701	0.106222	-0.049934	77.328
28.011	4.100573	-0.016855	1.511767	0.540138	0.106560	-0.050600	77.464
28.030	4.090573	-0.016899	1.517544	0.540422	0.106842	-0.051251	77.582
28.045	4.080572	-0.016813	1.522947	0.540566	0.107073	-0.051888	77.683
28.057	4.070572	-0.016609	1.528004	0.540580	0.107257	-0.052513	77.767
28.065	4.060572	-0.016298	1.532740	0.540476	0.107397	-0.053127	77.837
28.070	4.050572	-0.015887	1.537178	0.540262	0.107497	-0.053730	77.892
28.072	4.040572	-0.015385	1.541339	0.539947	0.107559	-0.054322	77.935
28.071	4.030572	-0.014799	1.545240	0.539538	0.107586	-0.054905	77.965

Table III - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 1.00$, $\mu = 0.300$. Normal pressure, $p = 12.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.521574	0.008252	0.008252	0.000856	0.000672	0.000672	0.336
.500	0.583586	0.008596	0.008983	0.000897	0.000673	0.000687	0.921
1.000	0.656741	0.009091	0.009947	0.000951	0.000674	0.000702	1.533
1.500	0.741808	0.009806	0.011249	0.001029	0.000676	0.000720	2.180
2.000	0.838433	0.010816	0.013027	0.001143	0.000679	0.000740	2.869
2.500	0.944901	0.012197	0.015433	0.001312	0.000684	0.000764	3.604
3.000	1.058458	0.013999	0.018613	0.001562	0.000691	0.000795	4.386
3.500	1.176059	0.016239	0.022681	0.001923	0.000701	0.000835	5.212
4.000	1.295044	0.013902	0.027715	0.002429	0.000716	0.000886	6.078
4.500	1.413472	0.021955	0.033760	0.003116	0.000733	0.000951	6.978
5.000	1.530103	0.025354	0.040840	0.004021	0.000760	0.001032	7.907
28.582	4.108265	-0.025822	1.565221	0.557478	0.112708	-0.053915	80.164
28.589	4.098265	-0.025366	1.569745	0.557257	0.112808	-0.054531	80.226
28.592	4.088265	-0.024813	1.573976	0.556930	0.112869	-0.055139	80.274
28.592	4.078265	-0.024172	1.577937	0.556504	0.112893	-0.055739	80.308
28.590	4.068265	-0.023451	1.581643	0.555988	0.112882	-0.056330	80.3[illegible]1

Table IV - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 1.00$, $\mu = 0.300$. Normal pressure, $p = 18.00\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,5}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{e_b{}^2}{h^2}$
0.	0.704259	0.012496	0.012496	0.001334	0.000968	0.000968	0.613
0.500	0.773978	0.013128	0.013706	0.001419	0.000970	0.000990	1.241
1.000	0.852085	0.013965	0.015241	0.001533	0.000972	0.001014	1.898
1.500	0.938231	0.015057	0.017200	0.001688	0.000976	0.001041	2.589
2.000	1.031473	0.016453	0.019688	0.001900	0.000981	0.001072	3.317
2.500	1.130381	0.018189	0.022809	0.002189	0.000989	0.001109	4.082
3.000	1.233292	0.020282	0.026658	0.002580	0.000999	0.001153	4.884
3.500	1.338575	0.022732	0.031311	0.003101	0.001013	0.001207	5.722
4.000	1.444828	0.025526	0.036825	0.003782	0.001031	0.001273	6.591
4.500	1.550963	0.028639	0.043241	0.004654	0.001053	0.001354	7.489
5.000	1.656205	0.032044	0.050589	0.005750	0.001080	0.001450	8.414
29.156	4.159187	-0.036514	1.600875	0.575991	0.118569	-0.054616	82.749
29.164	4.149187	-0.036003	1.605478	0.575758	0.118669	-0.055247	82.817
29.169	4.139187	-0.035390	1.609775	0.575413	0.118726	-0.055871	82.870
29.170	4.129187	-0.034684	1.613785	0.574966	0.118744	-0.056488	82.910
29.169	4.119187	-0.033893	1.617529	0.574424	0.118726	-0.057098	82.936

Table V- Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, a/b=1.00, $\mu = 0.300$. Normal pressure, $p = 24.50 \ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,5}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.887324	0.017832	0.017832	0.002001	0.001318	0.001318	0.975
0.500	0.959922	0.018778	0.019579	0.002157	0.001322	0.001349	1.641
1.000	1.038418	0.019949	0.021710	0.002359	0.001326	0.001383	2.336
1.500	1.122226	0.021376	0.024302	0.002619	0.001332	0.001421	3.060
2.000	1.210522	0.023084	0.027434	0.002956	0.001340	0.001465	3.817
2.500	1.302331	0.025090	0.031179	0.003389	0.001350	0.001516	4.604
3.000	1.396643	0.027398	0.035603	0.003942	0.001364	0.001575	5.421
3.500	1.492512	0.030004	0.040762	0.004640	0.001381	0.001645	6.268
4.000	1.589121	0.032895	0.046704	0.005510	0.001401	0.001727	7.141
4.500	1.685807	0.036054	0.053465	0.006581	0.001426	0.001822	8.040
5.000	1.782061	0.039465	0.061077	0.007881	0.001456	0.001934	8.962
29.845	4.216877	-0.049362	1.643312	0.597627	0.125538	-0.055255	85.849
29.854	4.206877	-0.048771	1.647968	0.597366	0.125629	-0.055903	85.923
29.860	4.196877	-0.048074	1.652300	0.596987	0.125677	-0.056545	85.981
29.862	4.186877	-0.047279	1.656330	0.596502	0.125683	-0.057182	86.024
29.861	4.176877	-0.046396	1.660081	0.595917	0.125651	-0.057814	86.052

Table VI - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, a/b = 1.00, μ = 0.300. Normal pressure, p = 30.00 Eh^4/b^4.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{1,3}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{3,3}}{h}$	$\frac{w_{1,5}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{eb^2}{h^2}$
0.	1.020802	0.022490	0.022490	0.002652	0.001615	0.001615	1.291
0.500	1.093572	0.023659	0.024643	0.002878	0.001620	0.001654	1.983
1.000	1.170845	0.025052	0.027203	0.003161	0.001626	0.001696	2.700
1.500	1.252071	0.026690	0.030234	0.003516	0.001633	0.001744	3.445
2.000	1.336590	0.028587	0.033798	0.003961	0.001643	0.001798	4.217
2.500	1.423682	0.030751	0.037955	0.004514	0.001656	0.001859	5.018
3.000	1.512635	0.033183	0.042753	0.005199	0.001671	0.001930	5.844
3.500	1.602790	0.035877	0.048255	0.006038	0.001690	0.002011	6.697
4.000	1.693571	0.038824	0.054486	0.007057	0.001713	0.002105	7.574
4.500	1.784501	0.042011	0.061486	0.008282	0.001740	0.002213	8.473
5.000	1.875199	0.045424	0.069285	0.009742	0.001772	0.002336	9.395
30.436	4.263910	-0.060373	1.679468	0.615745	0.131466	-0.055621	88.516
30.446	4.253910	-0.059704	1.684135	0.615448	0.131348	-0.056283	88.593
30.453	4.243909	-0.058926	1.688463	0.615031	0.131583	-0.056940	88.653
30.455	4.233909	-0.058047	1.692479	0.614503	0.131574	-0.057594	88.697
30.454	4.223909	-0.057077	1.696205	0.613874	0.131526	-0.058245	88.725

Table VII - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 4.00$, $\mu = 0.300$. Normal pressure, $p = 2.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.383	0.068	0.015	0.004	0.015
0.500	0.395	0.077	0.018	0.005	0.516
1.000	0.407	0.088	0.021	0.005	1.019
1.500	0.421	0.103	0.026	0.007	1.522
2.000	0.436	0.123	0.034	0.008	2.027
2.500	0.454	0.150	0.045	0.010	2.535
3.000	0.473	0.185	0.064	0.014	3.049
3.500	0.493	0.230	0.099	0.019	3.575
4.000	0.508	0.263	0.188	0.020	4.137
4.101	0.498	0.256	0.238	0.015	4.276
4.133	0.488	0.249	0.264	0.011	4.329
4.151	0.478	0.243	0.285	0.008	4.366
4.163	0.468	0.237	0.303	0.006	4.396
4.172	0.458	0.232	0.319	0.003	4.422
4.179	0.448	0.227	0.335	0.001	4.446
4.185	0.438	0.221	0.349	-0.001	4.469
4.190	0.428	0.216	0.364	-0.003	4.492
4.195	0.418	0.212	0.378	-0.005	4.514
4.199	0.408	0.207	0.391	-0.006	4.537
4.204	0.398	0.202	0.405	-0.008	4.561
4.209	0.388	0.197	0.419	-0.009	4.586
4.215	0.378	0.192	0.432	-0.011	4.612
4.221	0.368	0.188	0.446	-0.012	4.640
4.228	0.358	0.183	0.460	-0.013	4.670
4.236	0.348	0.178	0.474	-0.015	4.701
4.245	0.338	0.174	0.488	-0.016	4.736
4.256	0.328	0.169	0.503	-0.017	4.772
4.267	0.318	0.164	0.518	-0.018	4.812
4.280	0.308	0.160	0.533	-0.019	4.854
4.294	0.298	0.155	0.549	-0.019	4.901
4.310	0.288	0.150	0.566	-0.020	4.951
4.329	0.278	0.146	0.583	-0.021	5.006
4.349	0.268	0.141	0.601	-0.021	5.065
4.372	0.258	0.136	0.619	-0.022	5.130
4.397	0.248	0.131	0.638	-0.023	5.202
4.426	0.238	0.127	0.659	-0.023	5.280
4.458	0.228	0.122	0.680	-0.023	5.366
4.494	0.218	0.117	0.703	-0.024	5.461
4.534	0.208	0.113	0.727	-0.024	5.567

Table VIII - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, a/b = 4.00, μ = 0.300. Normal pressure, $p = 7.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	0.961	0.223	0.064	0.018	0.115
0.500	0.986	0.242	0.073	0.021	0.627
1.000	1.014	0.264	0.085	0.024	1.143
1.500	1.044	0.288	0.099	0.029	1.663
2.000	1.076	0.314	0.116	0.035	2.188
2.500	1.112	0.342	0.137	0.043	2.719
3.000	1.151	0.371	0.162	0.053	3.258
3.500	1.193	0.401	0.190	0.066	3.807
4.000	1.237	0.430	0.223	0.081	4.366
4.500	1.282	0.457	0.259	0.100	4.938
5.000	1.328	0.482	0.298	0.122	5.524
6.727	1.416	0.499	0.469	0.250	7.716
6.780	1.406	0.491	0.482	0.257	7.796
6.805	1.396	0.484	0.492	0.259	7.838
6.815	1.386	0.477	0.501	0.259	7.857
6.815	1.376	0.471	0.509	0.256	7.863

Table IX - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 4.00$, $\mu = 0.300$. Normal pressure, $p = 12.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	1.373	.352	.118	.037	.263
.500	1.404	.374	.132	.042	.789
1.000	1.437	.396	.147	.049	1.318
1.500	1.471	.420	.163	.056	1.852
2.000	1.508	.445	.182	.065	2.392
2.500	1.546	.470	.202	.075	2.937
3.000	1.587	.495	.225	.087	3.490
3.500	1.629	.521	.248	.100	4.050
4.000	1.672	.546	.274	.116	4.617
4.500	1.717	.571	.300	.134	5.194
5.00	1.761	.595	.327	.154	5.779
8.669	1.900	.659	.488	.491	10.617
8.675	1.890	.655	.486	.498	10.640
8.677	1.880	.651	.484	.504	10.658
8.677	1.870	.648	.483	.511	10.672
8.675	1.860	.644	.481	.517	10.684

Table X - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 4.00$, $\mu = 0.300$. Normal pressure, $p = 18.00\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{\epsilon b^2}{h^2}$
0.	1.723	0.464	0.171	0.059	0.448
0.500	1.755	0.485	0.186	0.066	0.984
1.000	1.789	0.507	0.202	0.074	1.524
1.500	1.824	0.529	0.219	0.082	2.068
2.000	1.861	0.552	0.237	0.092	2.618
2.500	1.899	0.575	0.256	0.103	3.173
3.000	1.938	0.597	0.276	0.115	3.733
3.500	1.978	0.620	0.296	0.129	4.300
4.000	2.018	0.643	0.318	0.144	4.873
4.500	2.059	0.665	0.340	0.160	5.452
5.000	2.101	0.687	0.362	0.178	6.039
10.247	2.305	0.795	0.544	0.578	12.930
10.254	2.295	0.792	0.543	0.585	12.956
10.258	2.285	0.789	0.541	0.591	12.977
10.260	2.275	0.785	0.539	0.597	12.995
10.258	2.265	0.782	0.536	0.603	13.009

Table XI - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, $a/b = 4.00$, $\mu = 0.300$. Normal pressure, $p = 24.50\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	2.053	0.569	0.224	0.083	0.672
0.500	2.085	0.589	0.238	0.091	1.216
1.000	2.118	0.610	0.254	0.099	1.765
1.500	2.153	0.630	0.270	0.109	2.318
2.000	2.188	0.651	0.287	0.119	2.875
2.500	2.224	0.671	0.304	0.130	3.433
3.000	2.261	0.692	0.322	0.142	4.003
3.500	2.298	0.713	0.341	0.155	4.574
4.000	2.336	0.733	0.360	0.169	5.150
4.500	2.374	0.753	0.379	0.184	5.732
5.000	2.412	0.773	0.398	0.199	6.319
11.964	2.675	0.916	0.607	0.659	15.448
11.973	2.665	0.913	0.605	0.665	15.478
11.978	2.655	0.910	0.603	0.672	15.503
11.981	2.645	0.907	0.601	0.678	15.523
11.981	2.635	0.904	0.599	0.683	15.540

Table XII - Values of deflection coefficients for various values of axial compressive load in the x-direction, P, for simply supported rectangular plate, a/b = 4.00, μ = 0.300. Normal pressure, $p = 30.00\ Eh^4/b^4$.

$\frac{Pb}{Eh^3}$	$\frac{w_{1,1}}{h}$	$\frac{w_{3,1}}{h}$	$\frac{w_{5,1}}{h}$	$\frac{w_{7,1}}{h}$	$\frac{eb^2}{h^2}$
0.	2.286	0.643	0.261	0.101	0.860
0.500	2.318	0.662	0.275	0.110	1.409
1.000	2.350	0.681	0.290	0.118	1.963
1.500	2.384	0.701	0.306	0.128	2.520
2.000	2.418	0.720	0.322	0.138	3.082
2.500	2.452	0.739	0.338	0.149	3.647
3.000	2.487	0.759	0.355	0.161	4.217
3.500	2.522	0.778	0.372	0.173	4.791
4.000	2.558	0.797	0.390	0.186	5.369
4.500	2.594	0.816	0.407	0.200	5.952
5.000	2.630	0.835	0.425	0.215	6.539
13.331	2.920	0.996	0.651	0.724	17.476
13.337	2.910	0.993	0.649	0.730	17.502
13.339	2.900	0.990	0.648	0.736	17.524
13.339	2.890	0.987	0.646	0.742	17.542
13.337	2.880	0.984	0.643	0.747	17.556

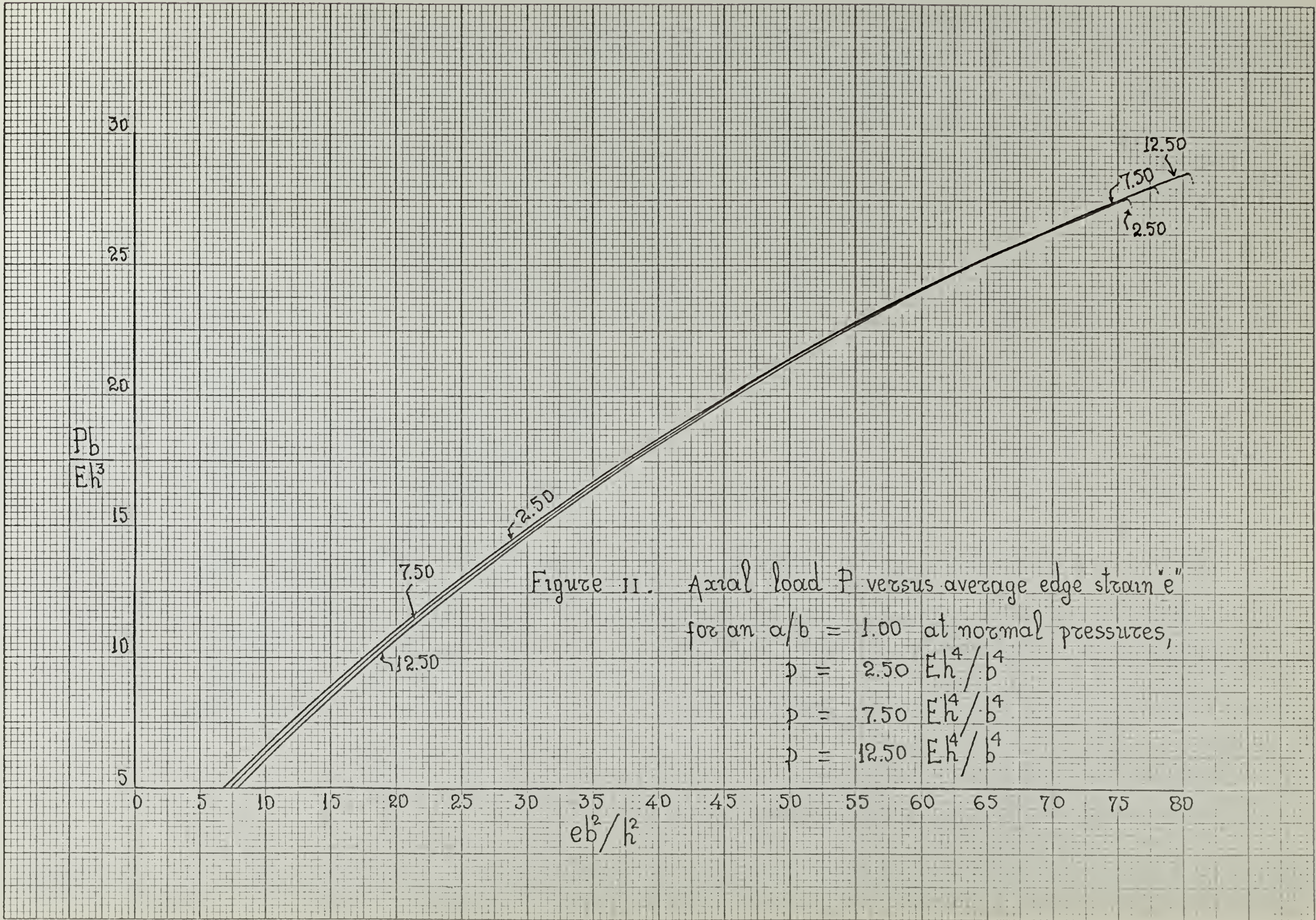

Figure 11. Axial load P versus average edge strain "e" for an $a/b = 1.00$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

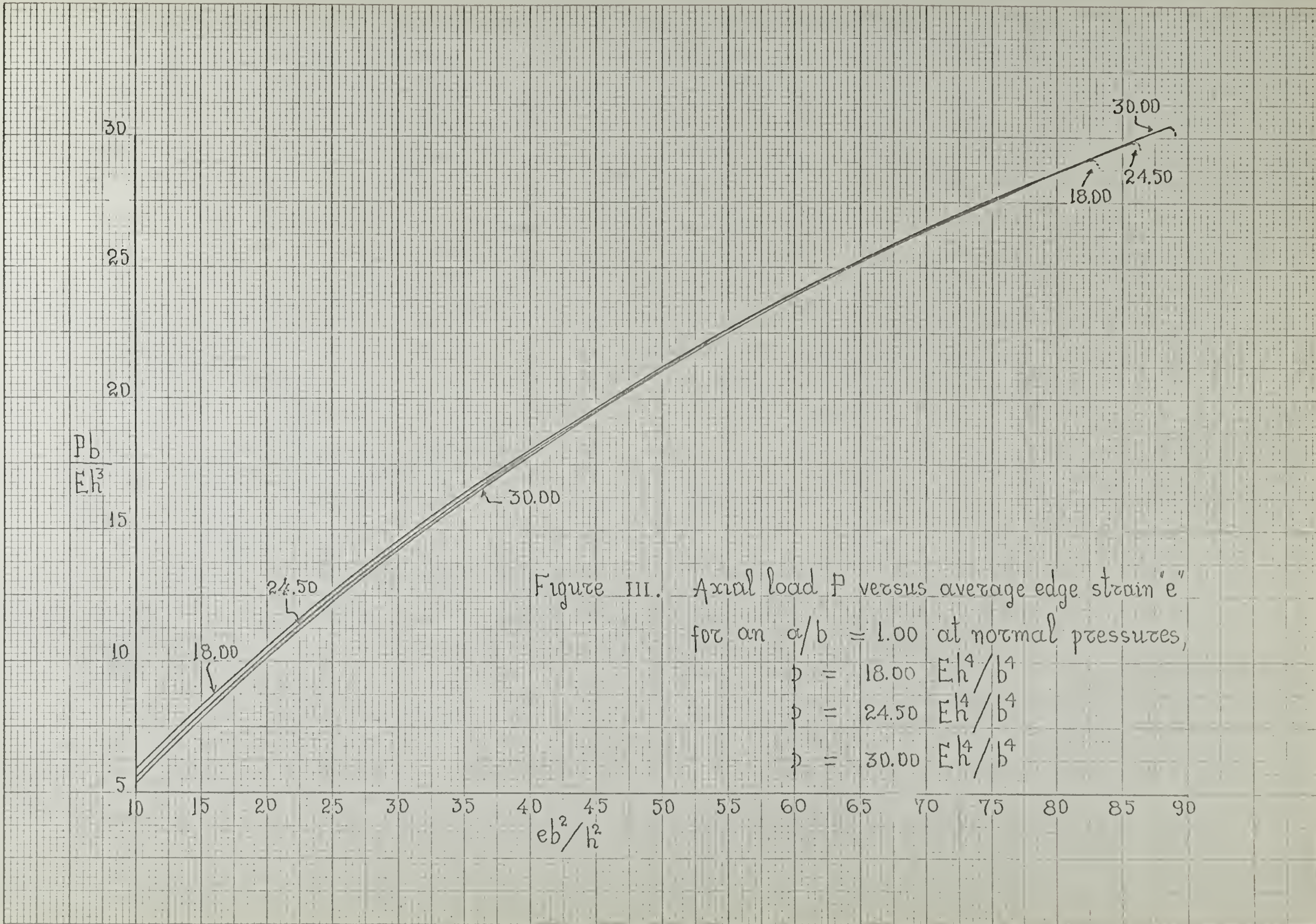

Figure III. Axial load P versus average edge strain "e" for an $a/b = 1.00$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

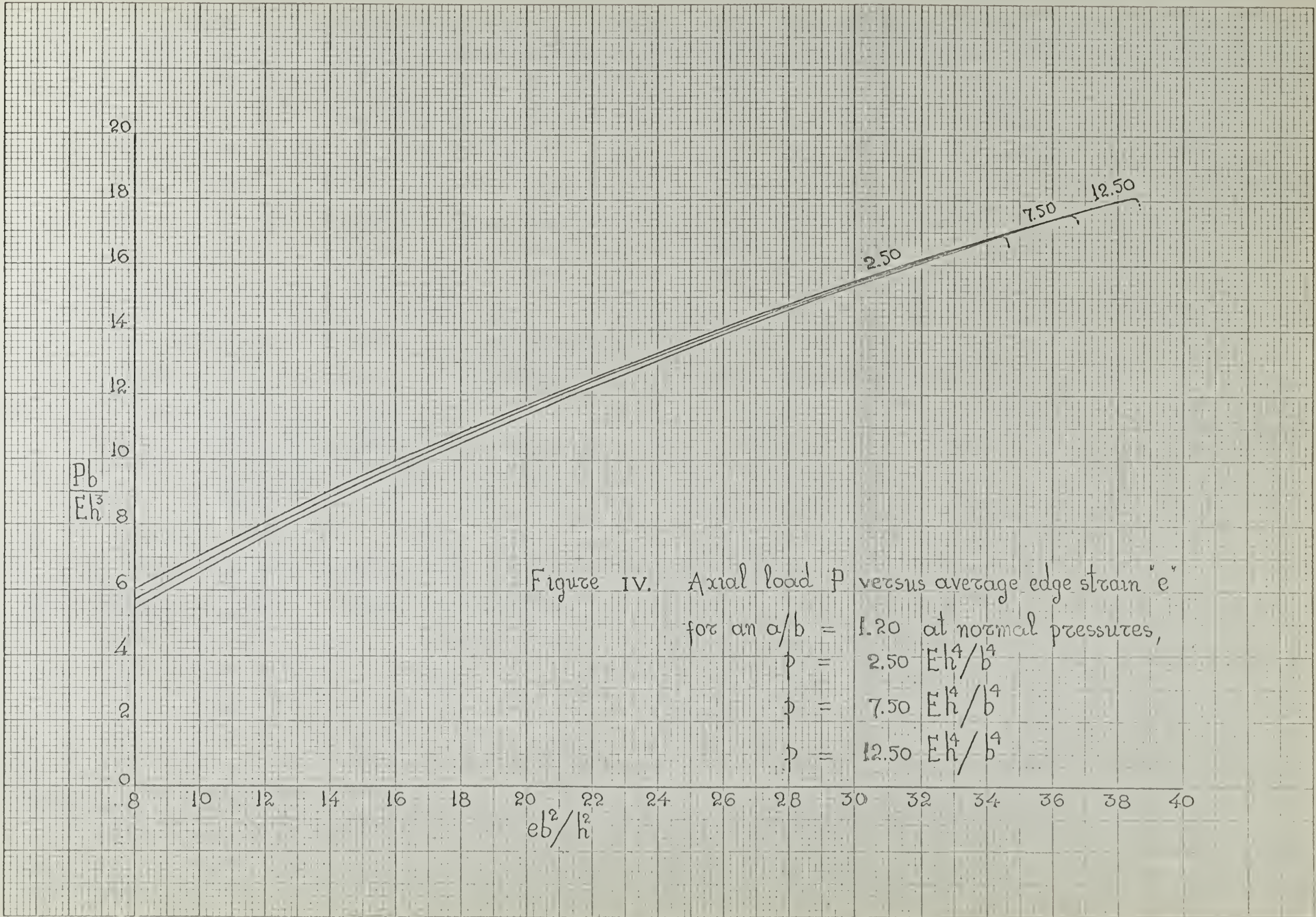

Figure IV. Axial load P versus average edge strain "e" for an $a/b = 1.20$ at normal pressures,
$p = 2.50\ Eh^4/b^4$
$p = 7.50\ Eh^4/b^4$
$p = 12.50\ Eh^4/b^4$

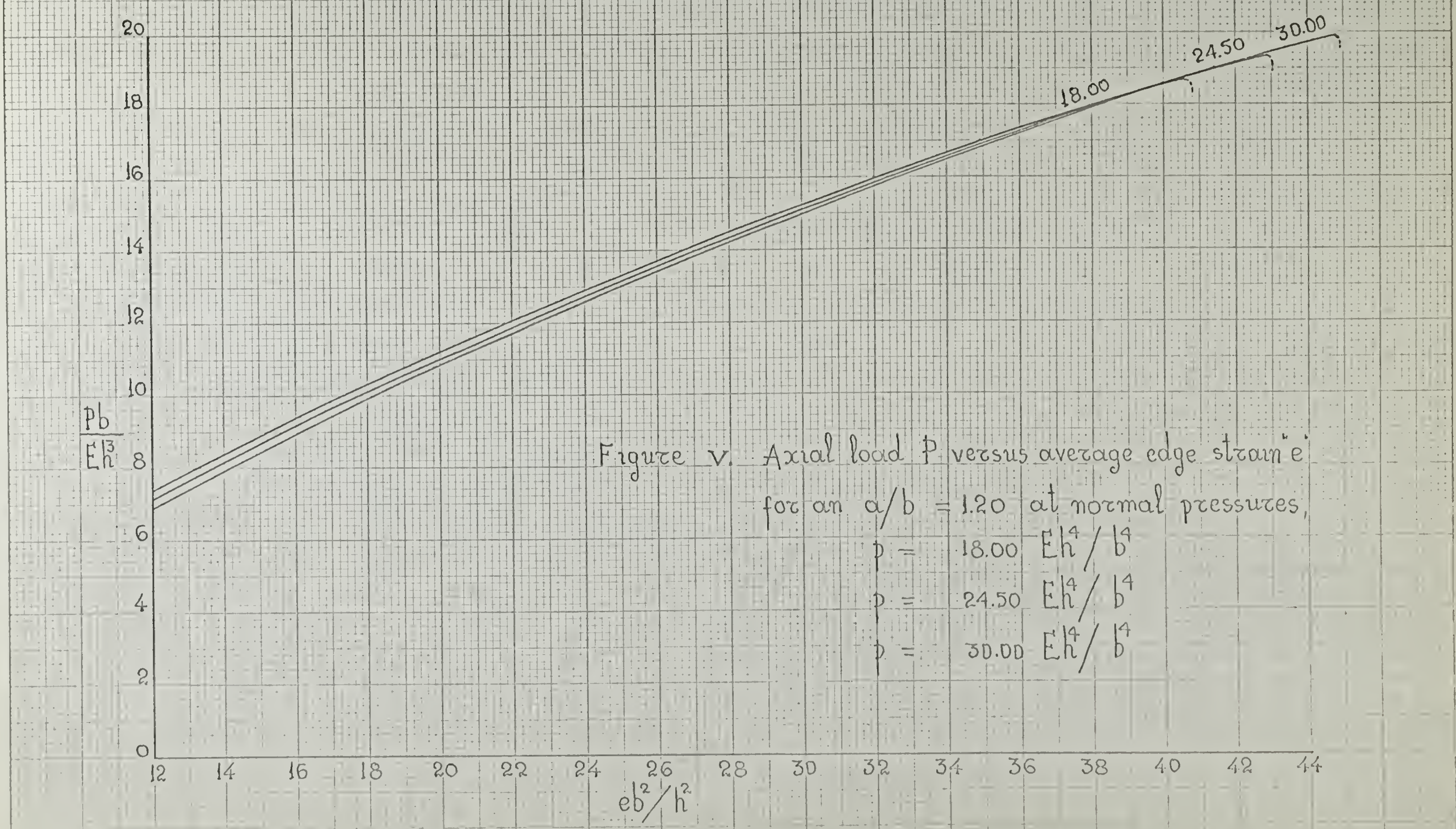

Figure V. Axial load P versus average edge strain "e" for an $a/b = 1.20$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

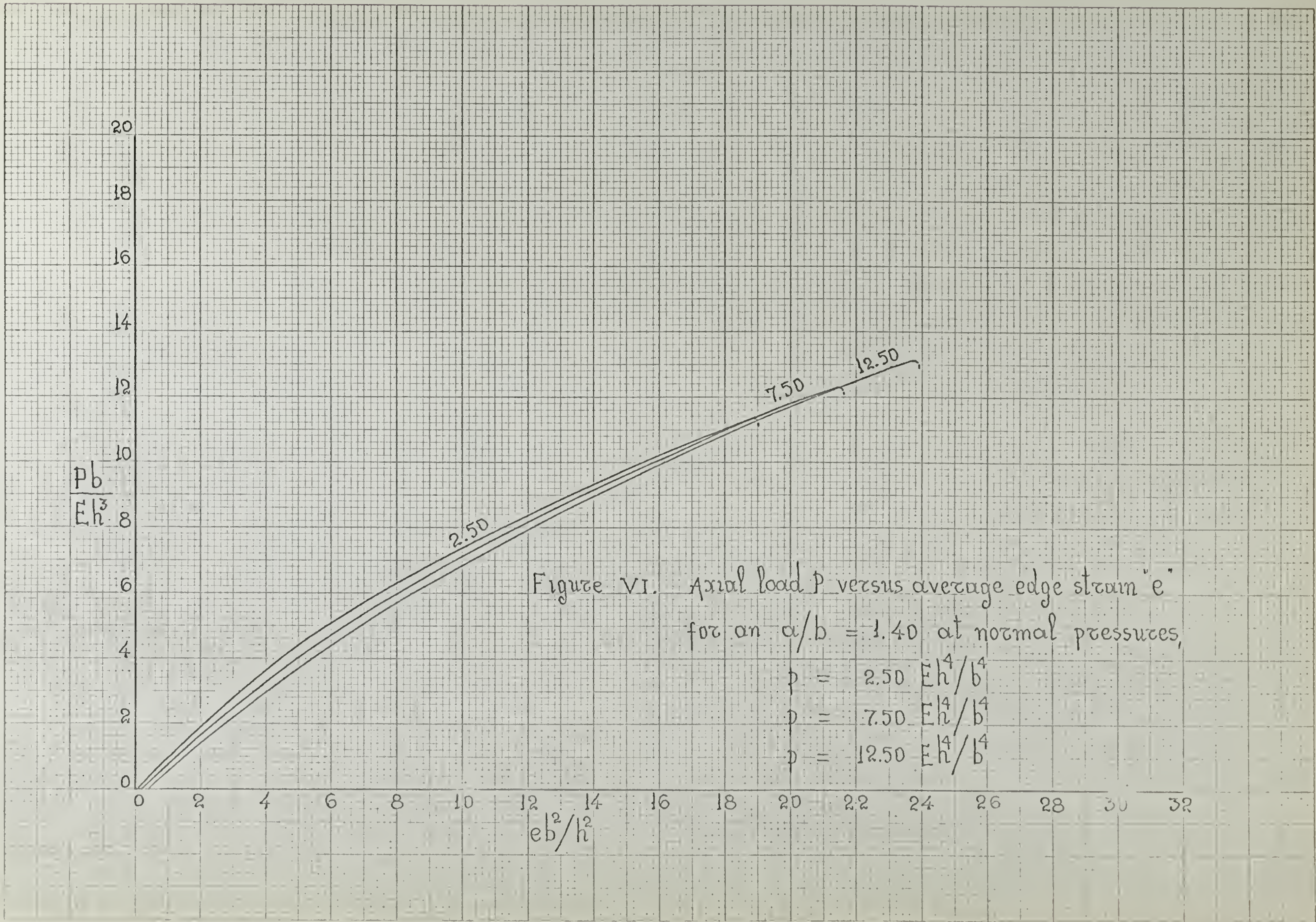

Figure VI. Axial load P versus average edge strain "e" for an $a/b = 1.40$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

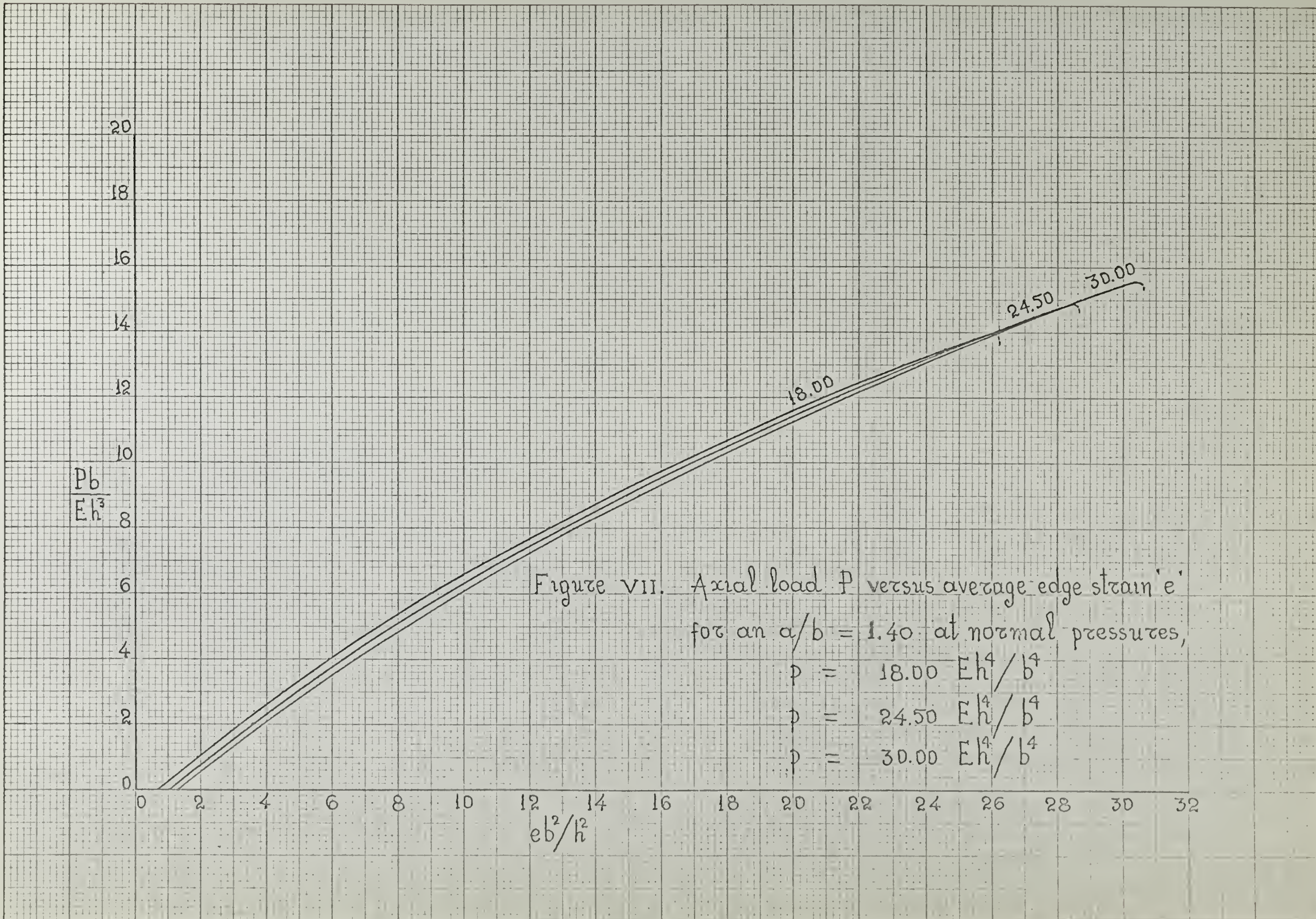

Figure VII. Axial load P versus average edge strain e' for an a/b = 1.40 at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

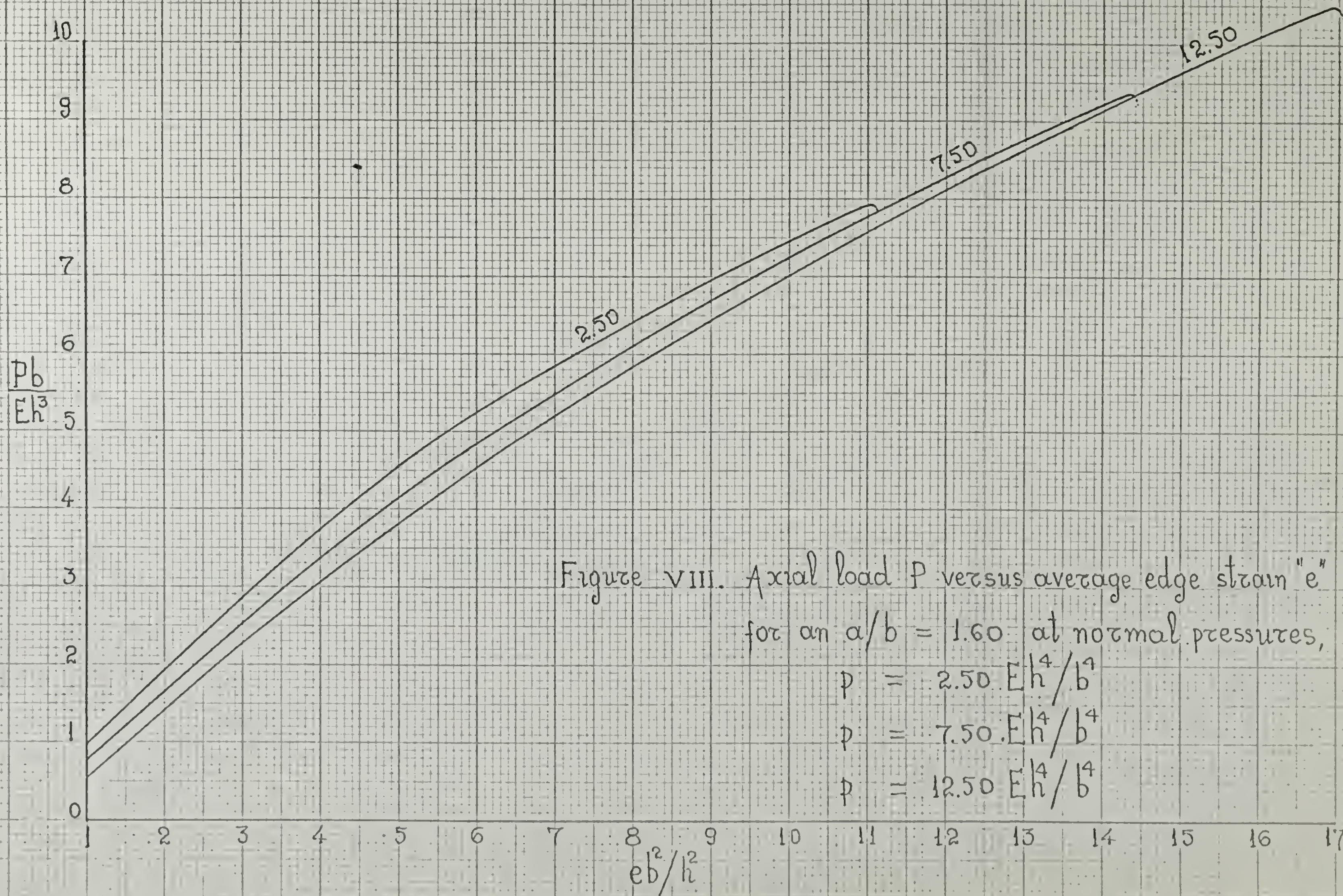

Figure VIII. Axial load P versus average edge strain "e" for an $a/b = 1.60$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

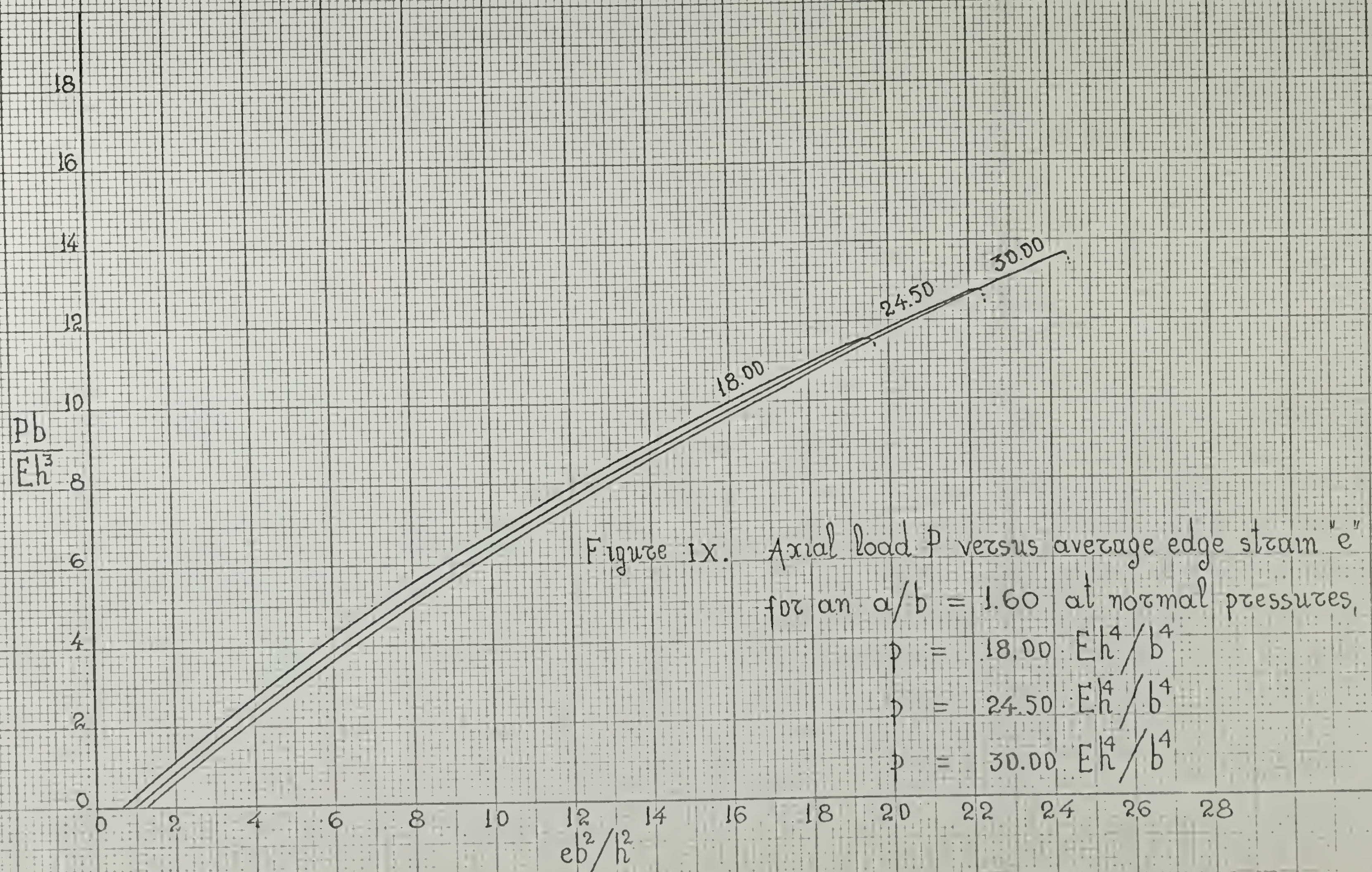

Figure IX. Axial load P versus average edge strain "e" for an $a/b = 1.60$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

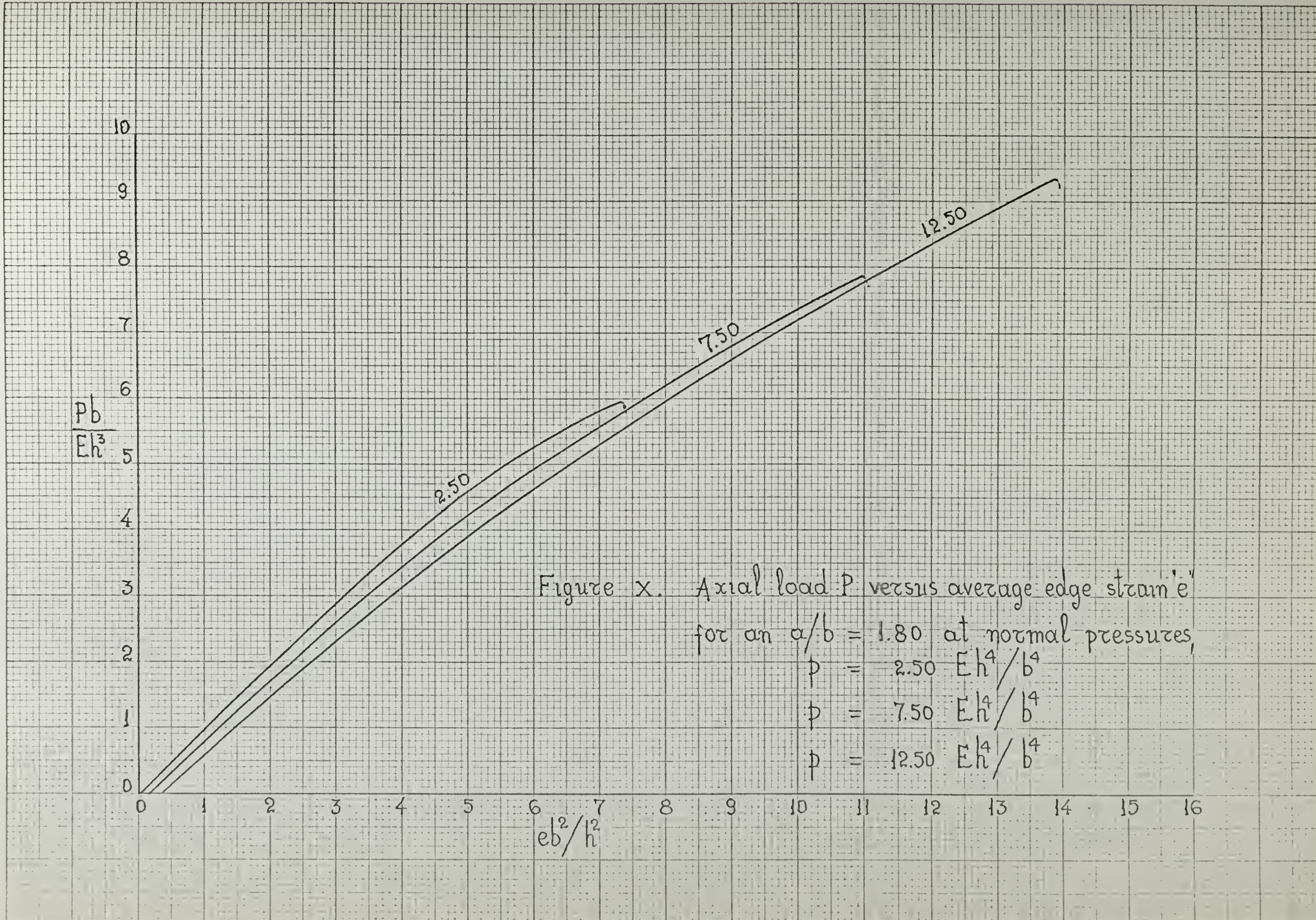

Figure X. Axial load P versus average edge strain 'e' for an $a/b = 1.80$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

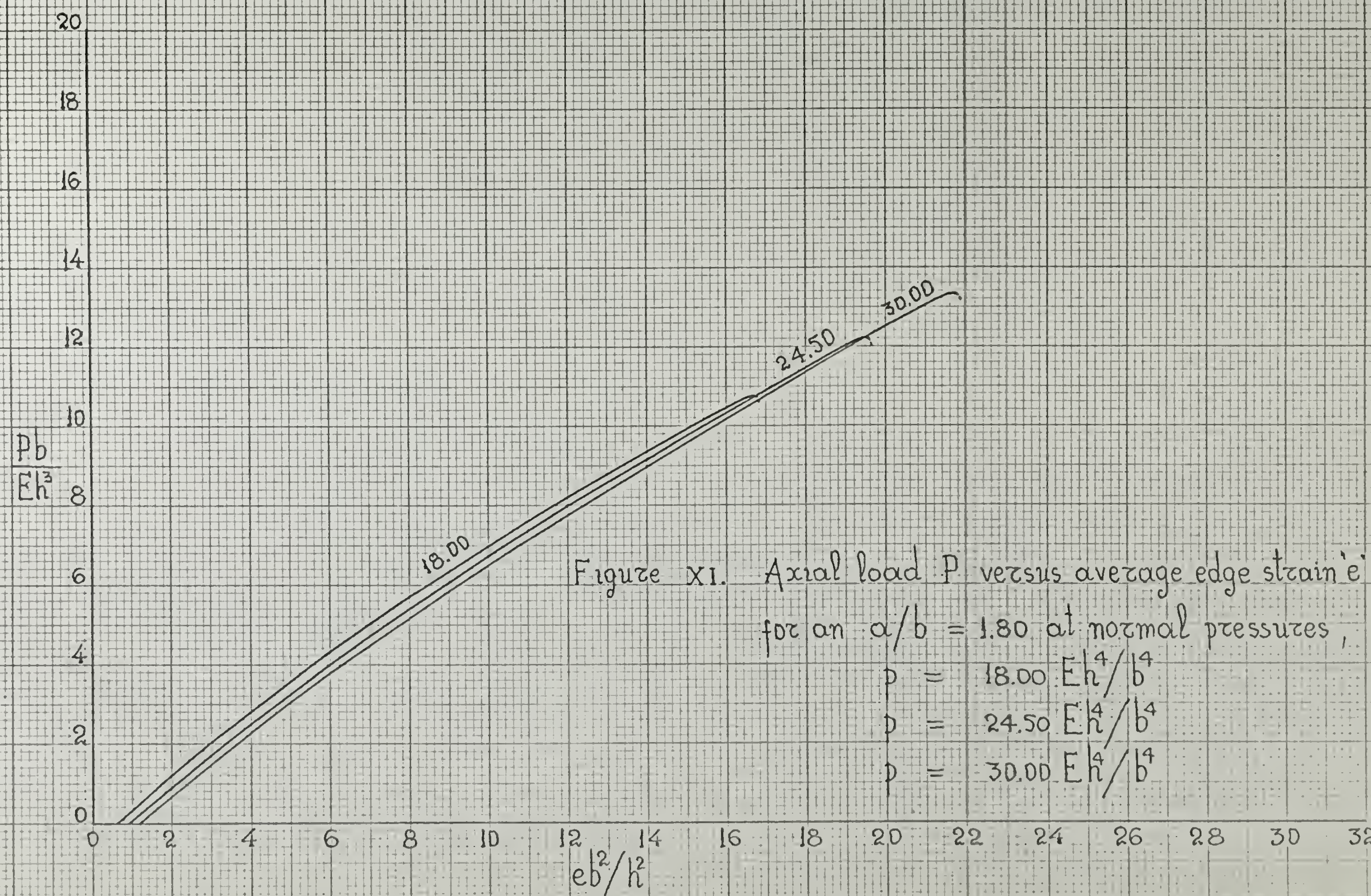

Figure XI. Axial load P versus average edge strain 'e' for an $a/b = 1.80$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

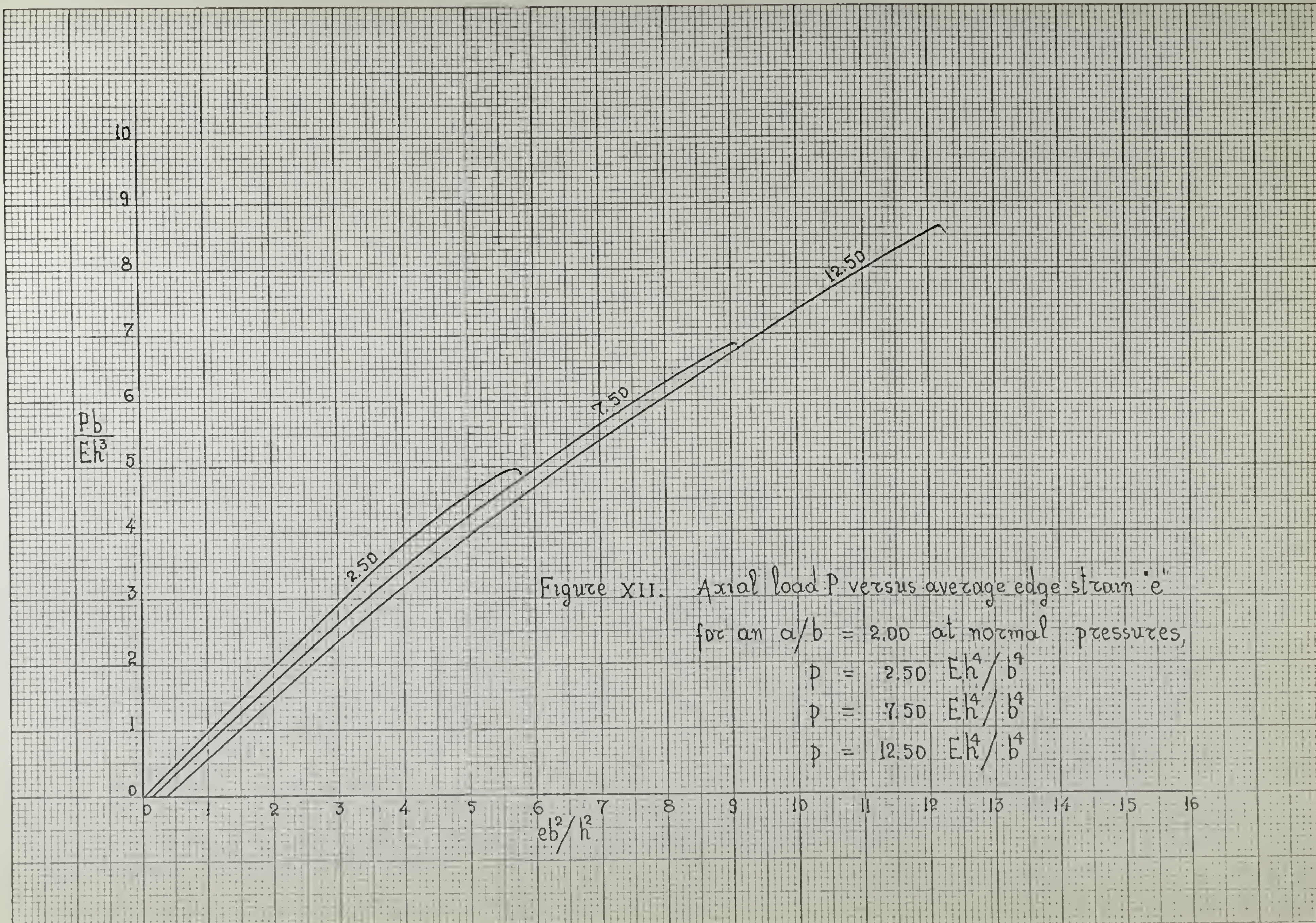

Figure XII. Axial load P versus average edge strain "e" for an $a/b = 2.00$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

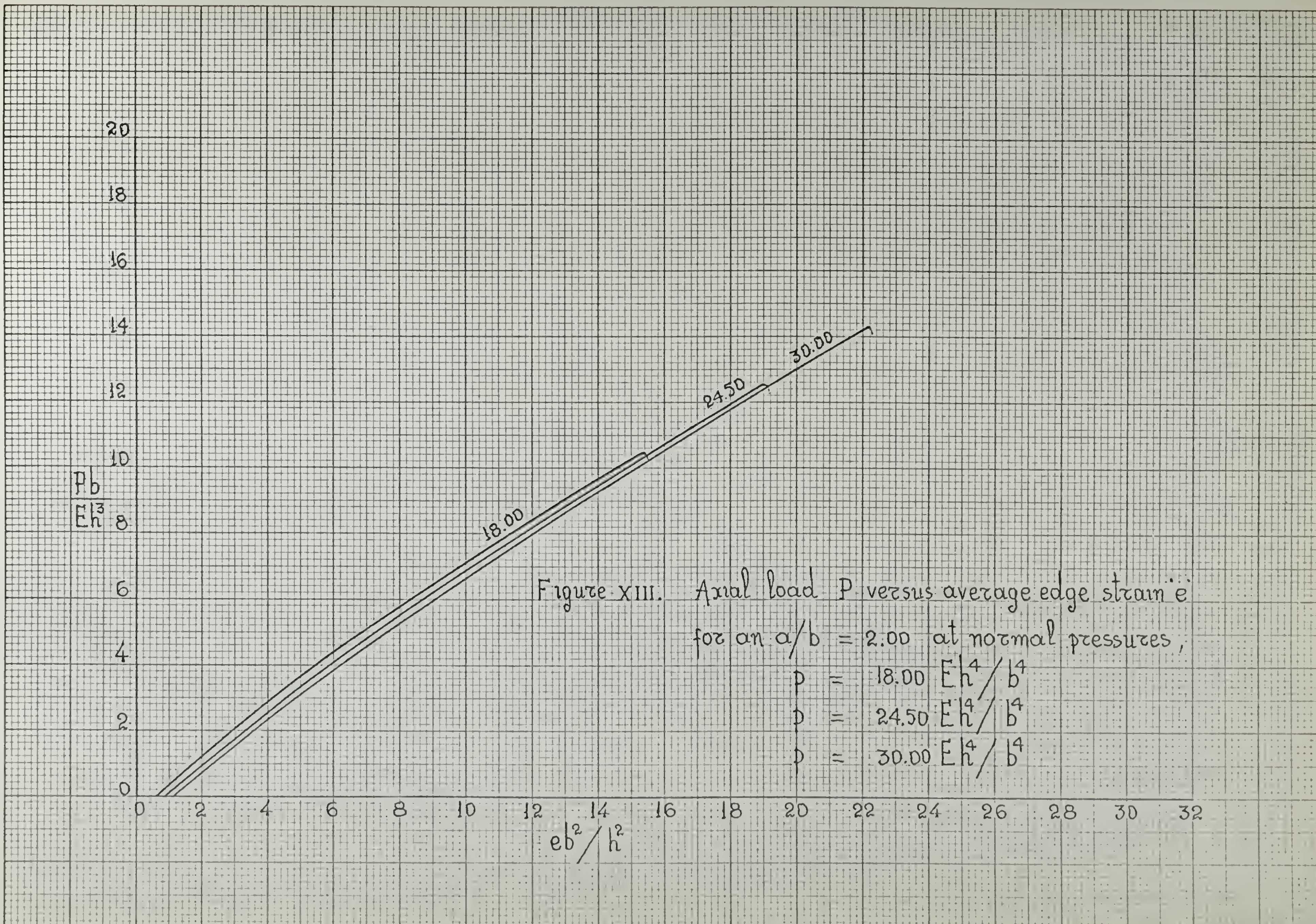

Figure XIII. Axial load P versus average edge strain e for an a/b = 2.00 at normal pressures,
$p = 18.00\ Eh^4/b^4$
$p = 24.50\ Eh^4/b^4$
$p = 30.00\ Eh^4/b^4$

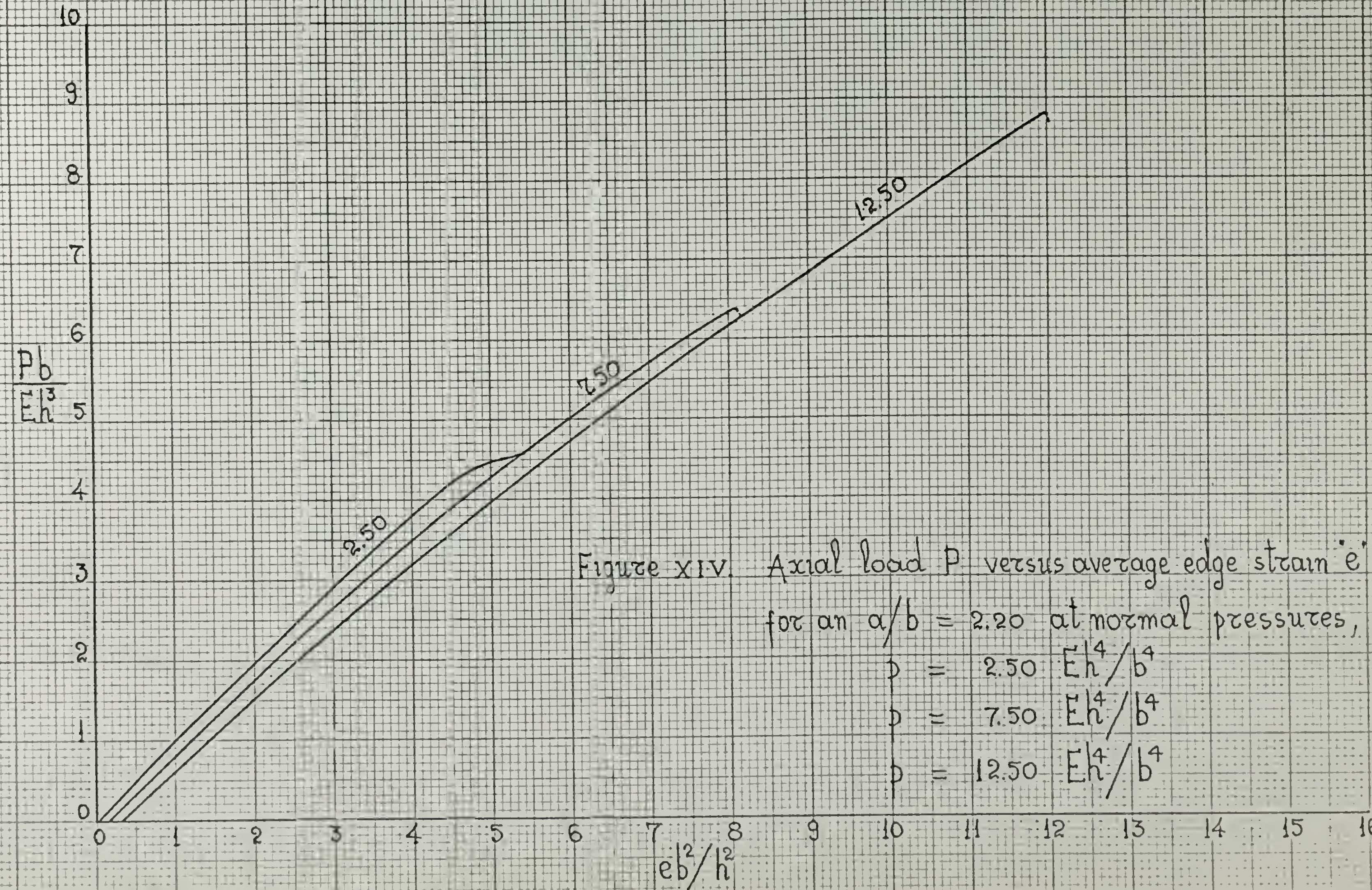

Figure XIV. Axial load P versus average edge strain "e" for an $a/b = 2.20$ at normal pressures,

$p = 2.50 \; Eh^4/b^4$

$p = 7.50 \; Eh^4/b^4$

$p = 12.50 \; Eh^4/b^4$

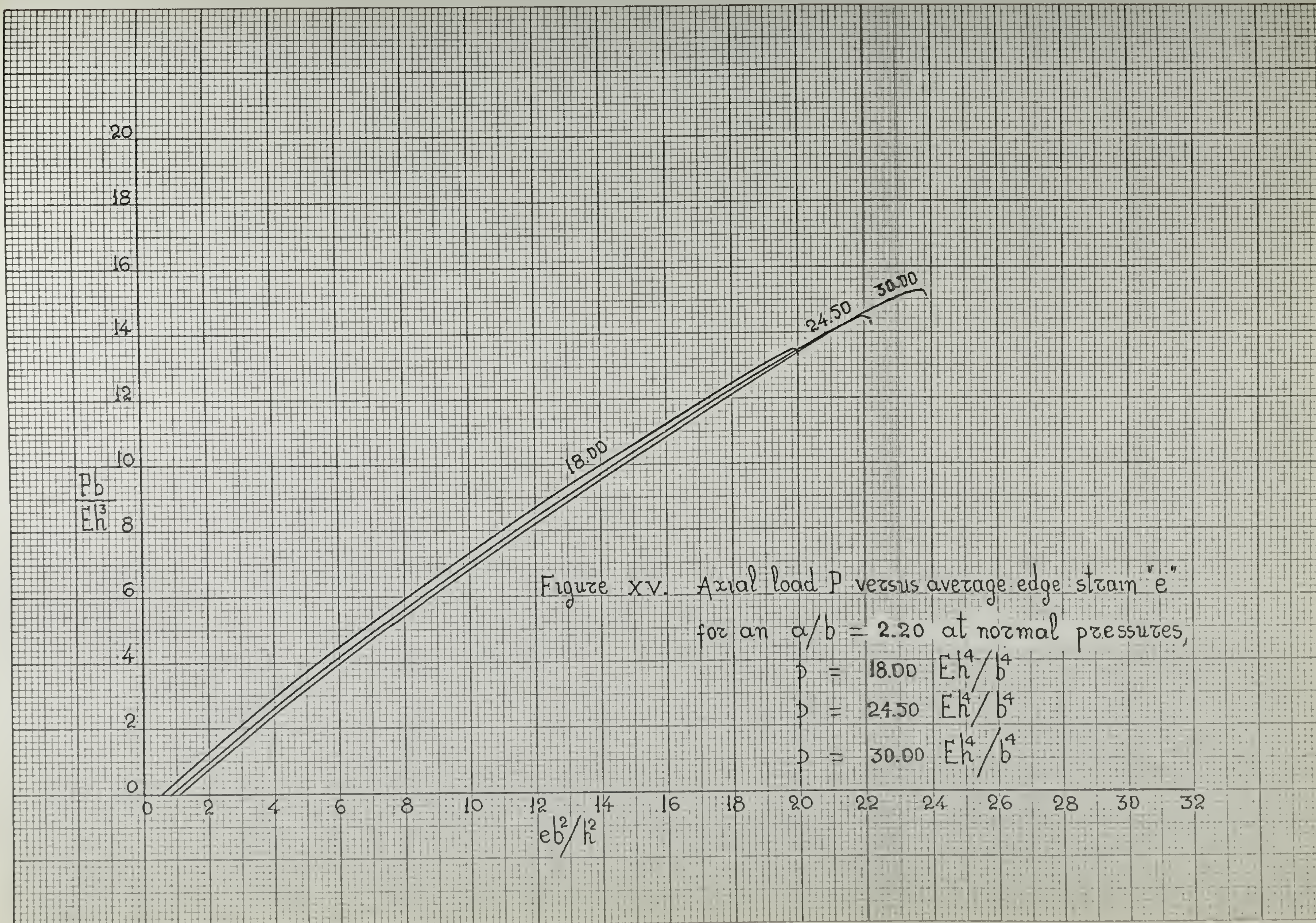

Figure XV. Axial load P versus average edge strain "e" for an $a/b = 2.20$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

20 Squares to the Inch

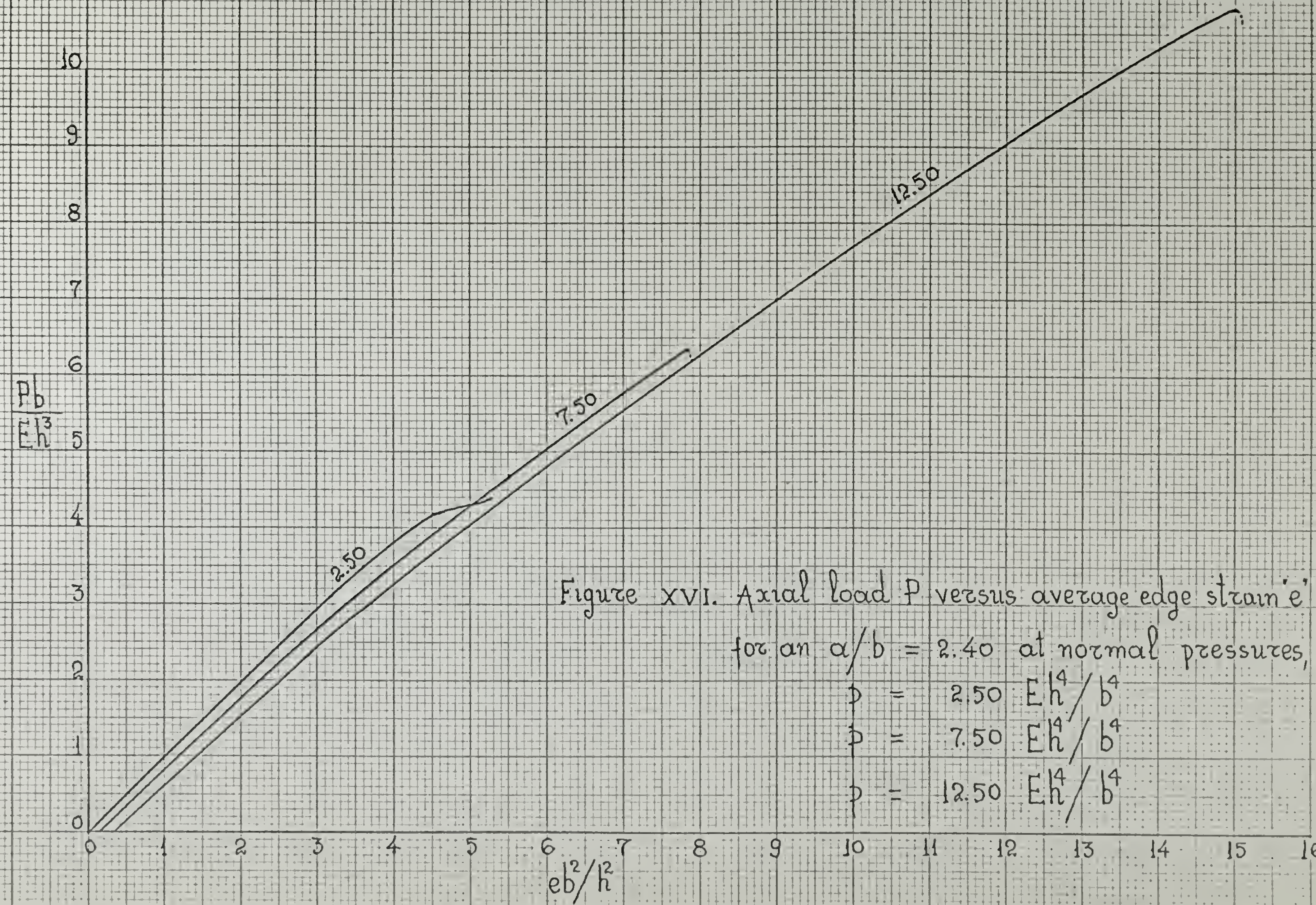

Figure XVI. Axial load P versus average edge strain 'e' for an $a/b = 2.40$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

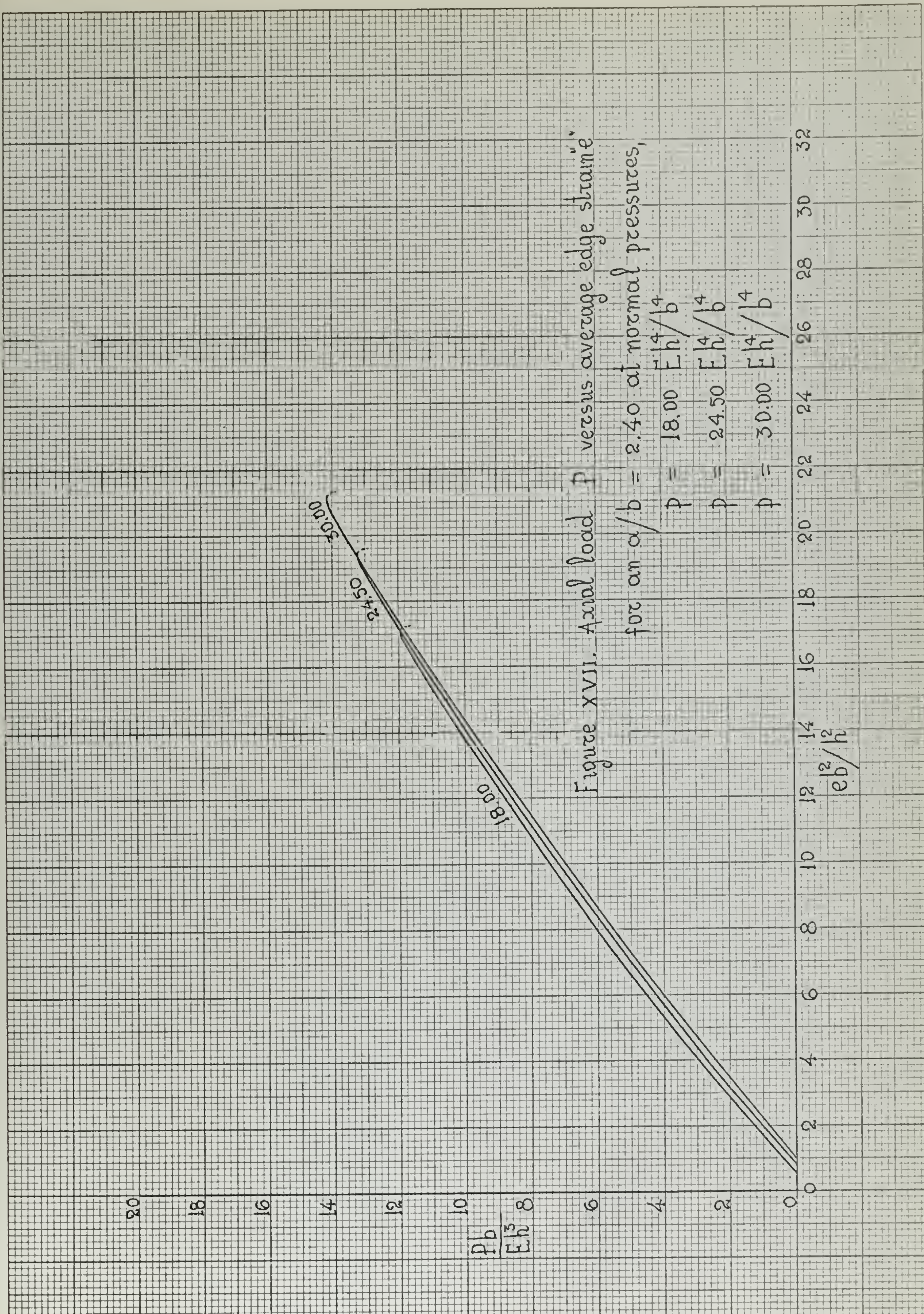

Figure XVII. Axial load P versus average edge strains for an $a/b = 2.40$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

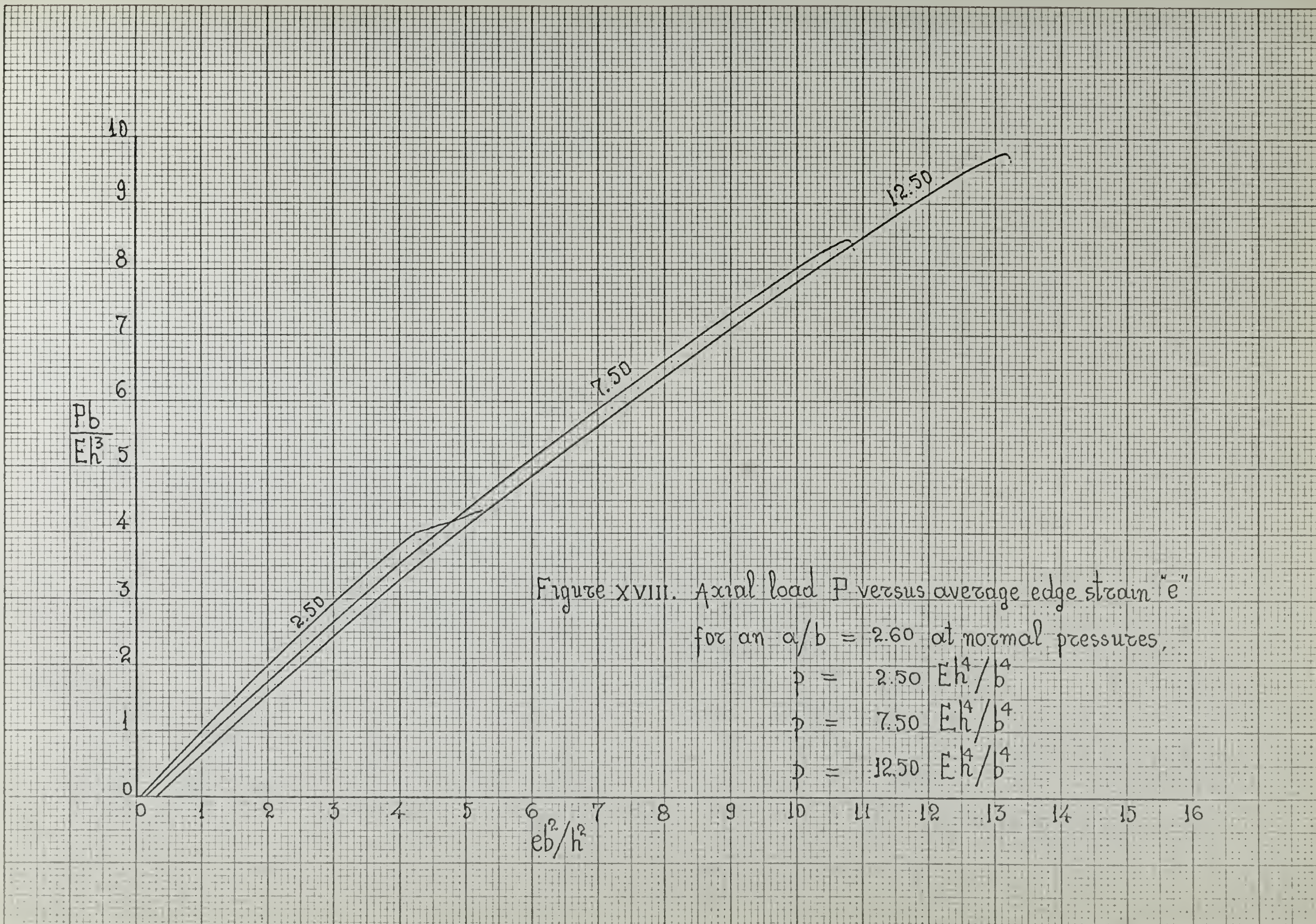

Figure XVIII. Axial load P versus average edge strain "e" for an $a/b = 2.60$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

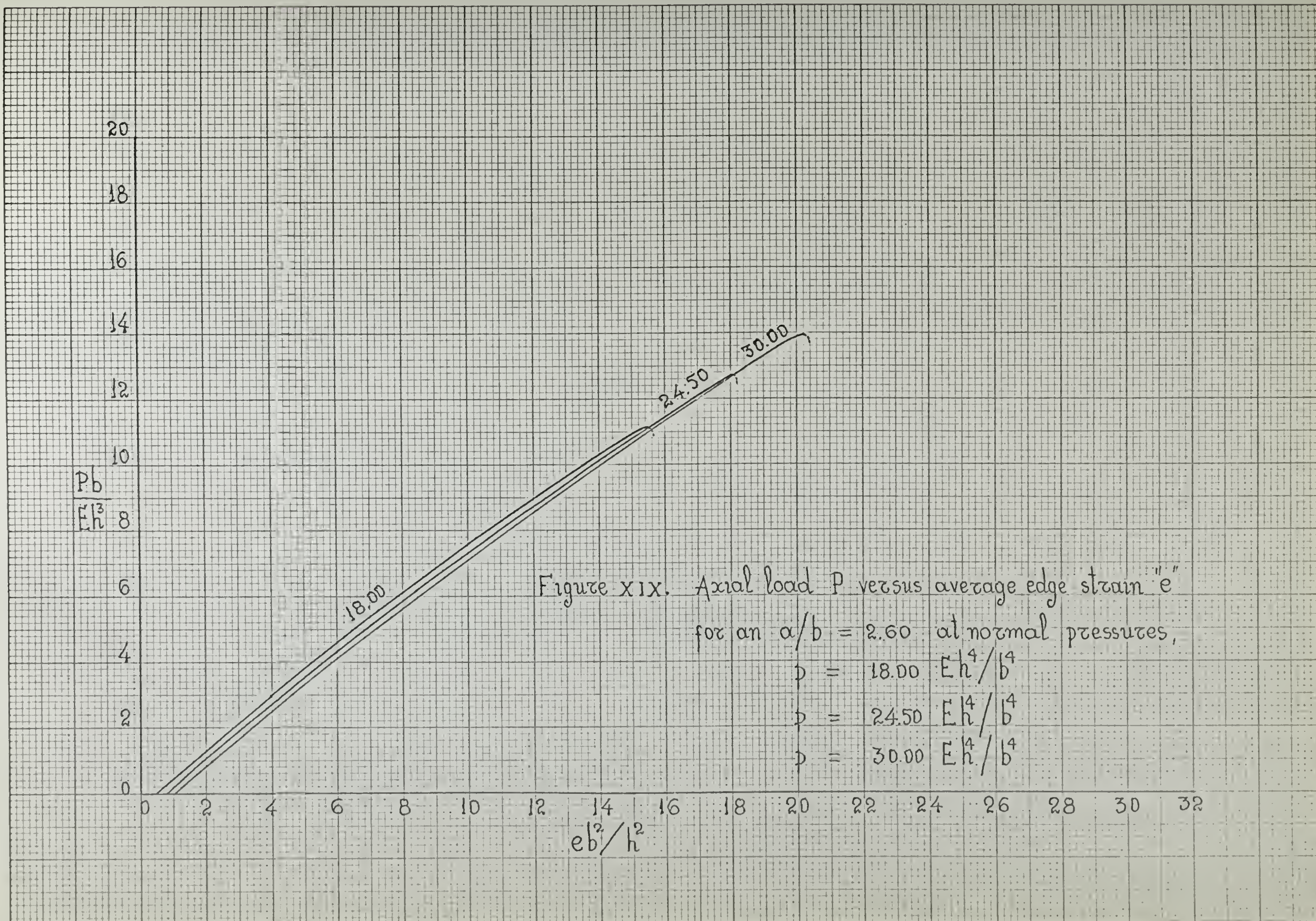

Figure XIX. Axial load P versus average edge strain "e" for an $a/b = 2.60$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

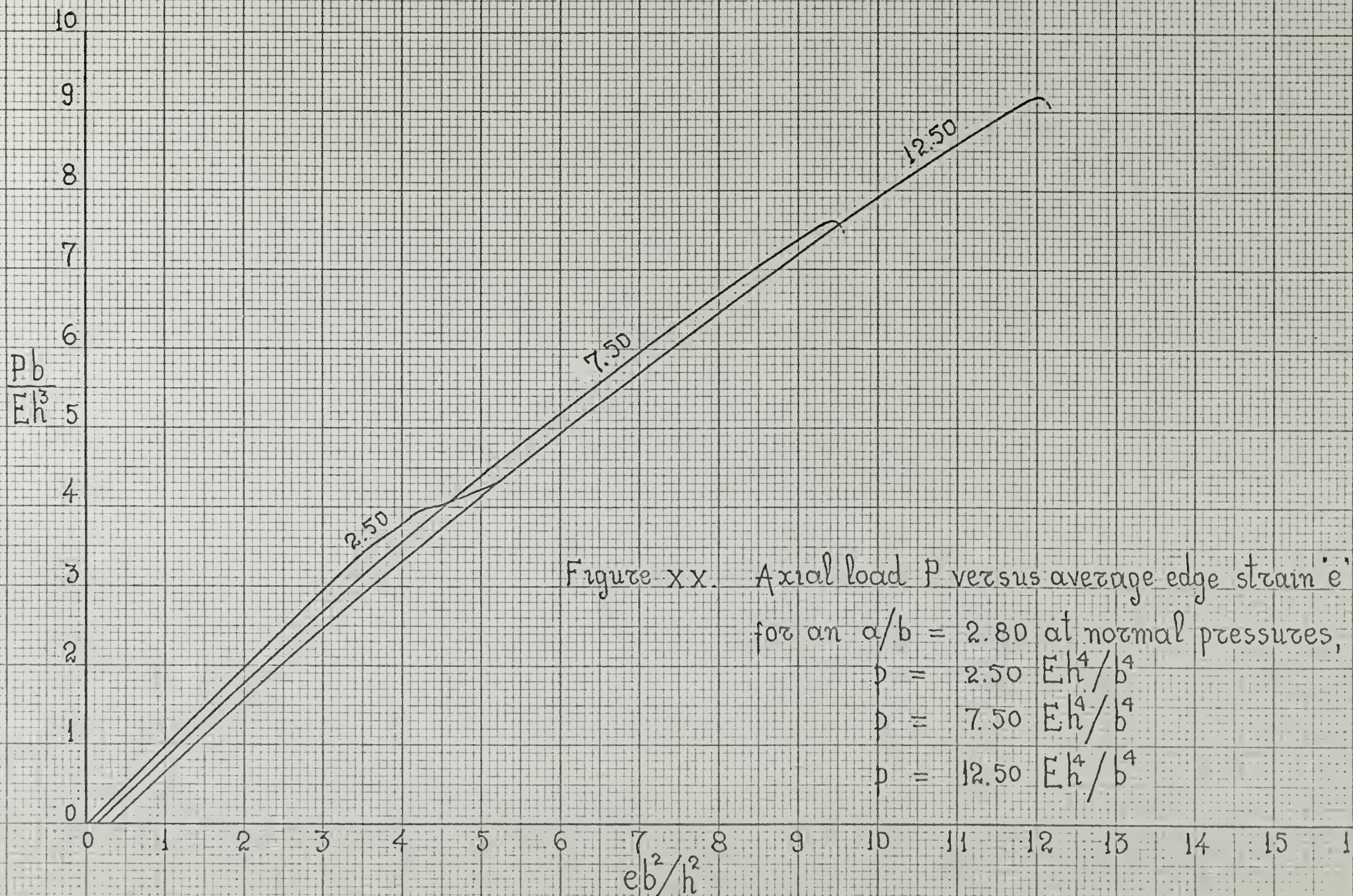

Figure XX. Axial load P versus average edge strain "e" for an $a/b = 2.80$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

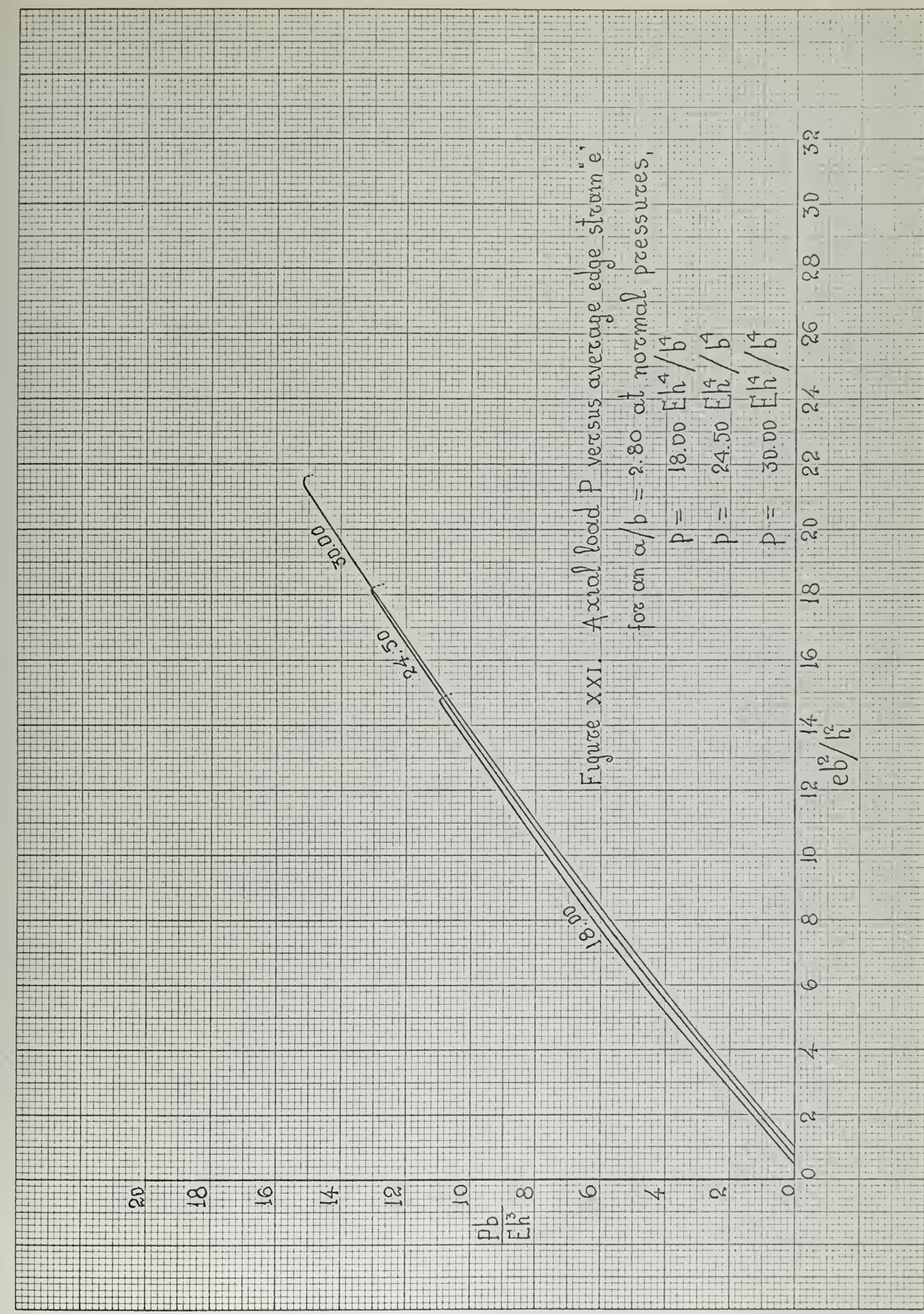

Figure XXI. Axial load P versus average edge strain "e" for an $a/b = 2.80$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

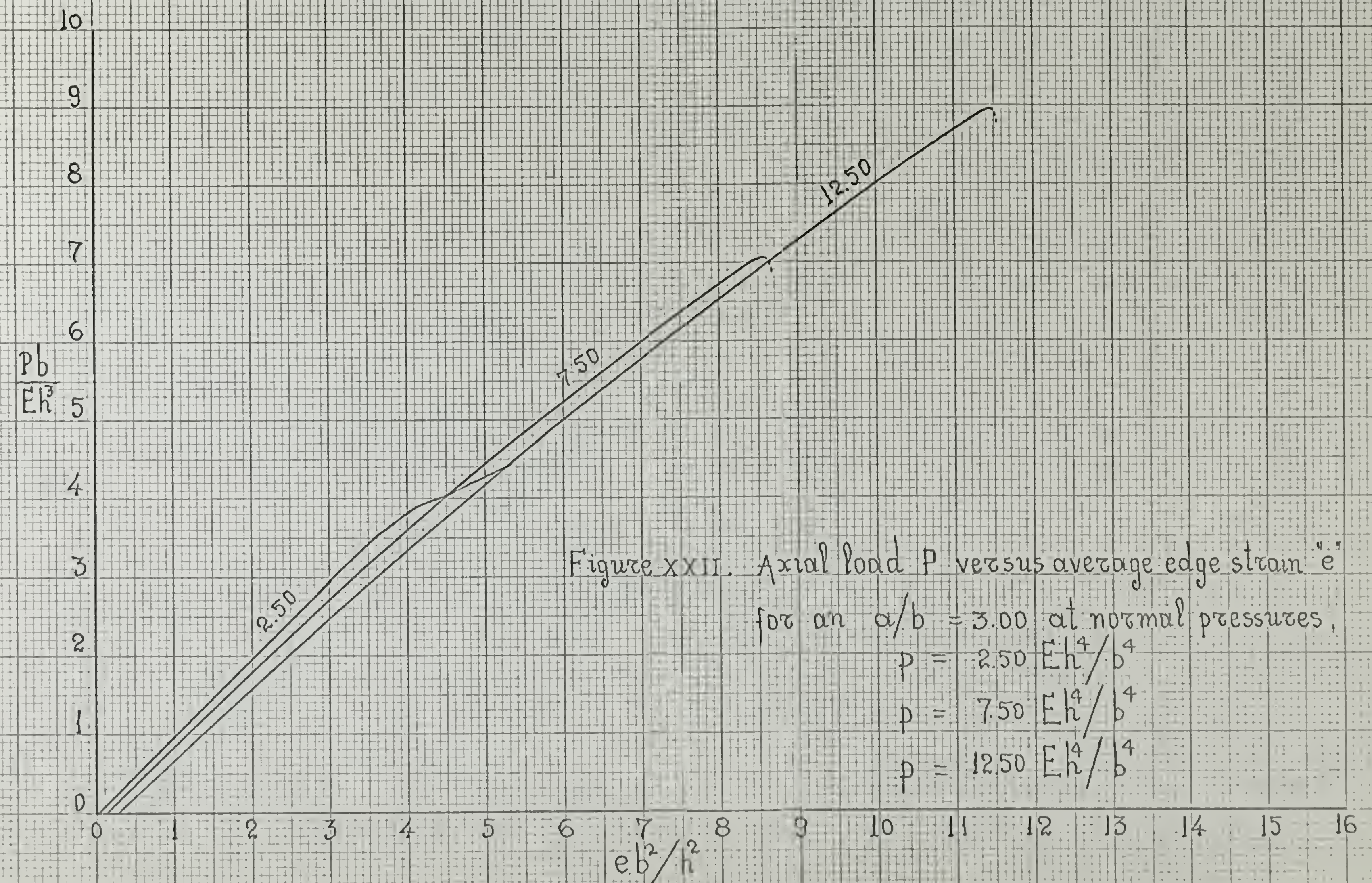

Figure XXII. Axial load P versus average edge strain "e" for an $a/b = 3.00$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

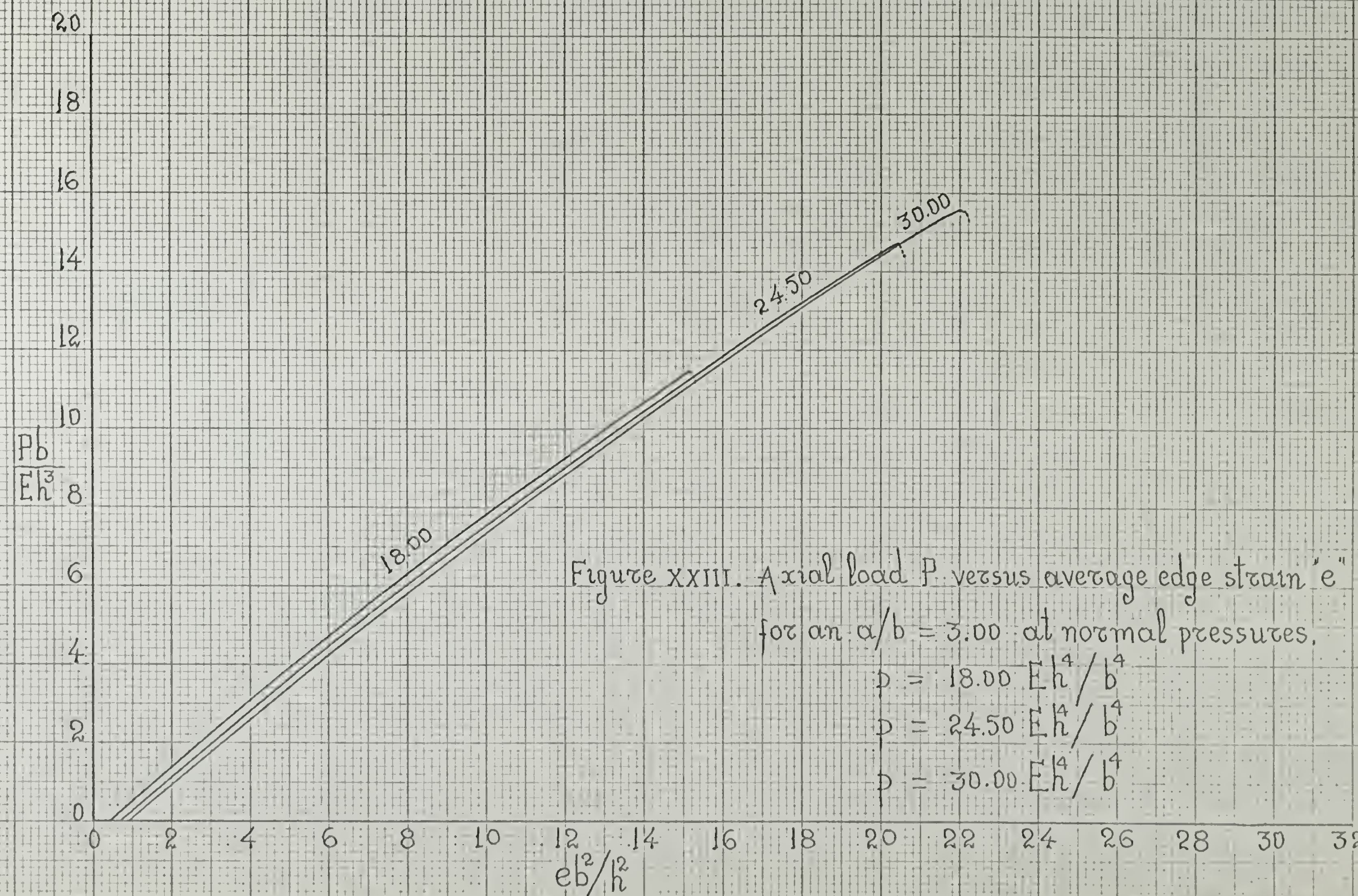

Figure XXIII. Axial load P versus average edge strain "e" for an $a/b = 3.00$ at normal pressures.

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

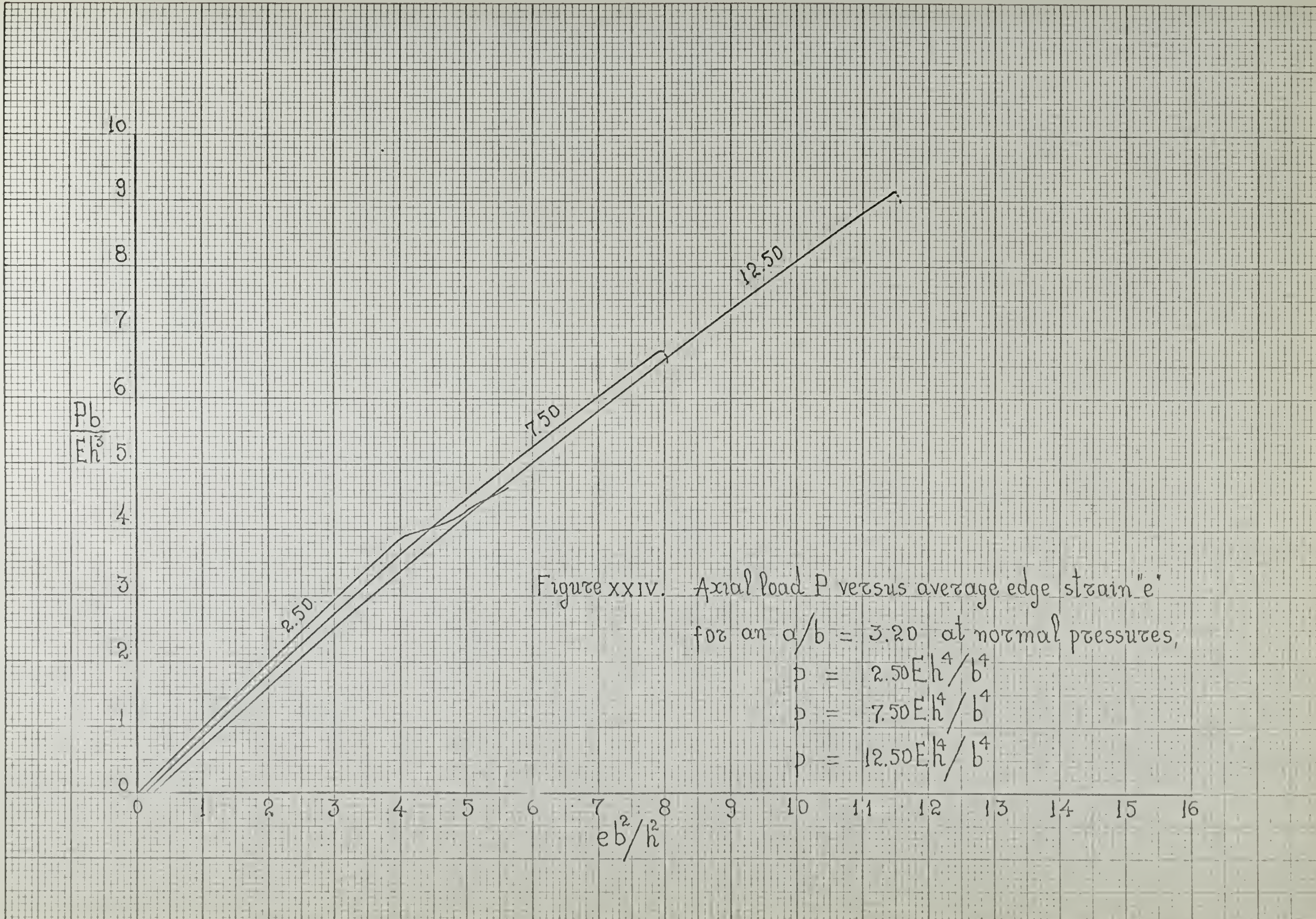

Figure XXIV. Axial load P versus average edge strain "e"
for an $a/b = 3.20$ at normal pressures,
$p = 2.50 Eh^4/b^4$
$p = 7.50 Eh^4/b^4$
$p = 12.50 Eh^4/b^4$

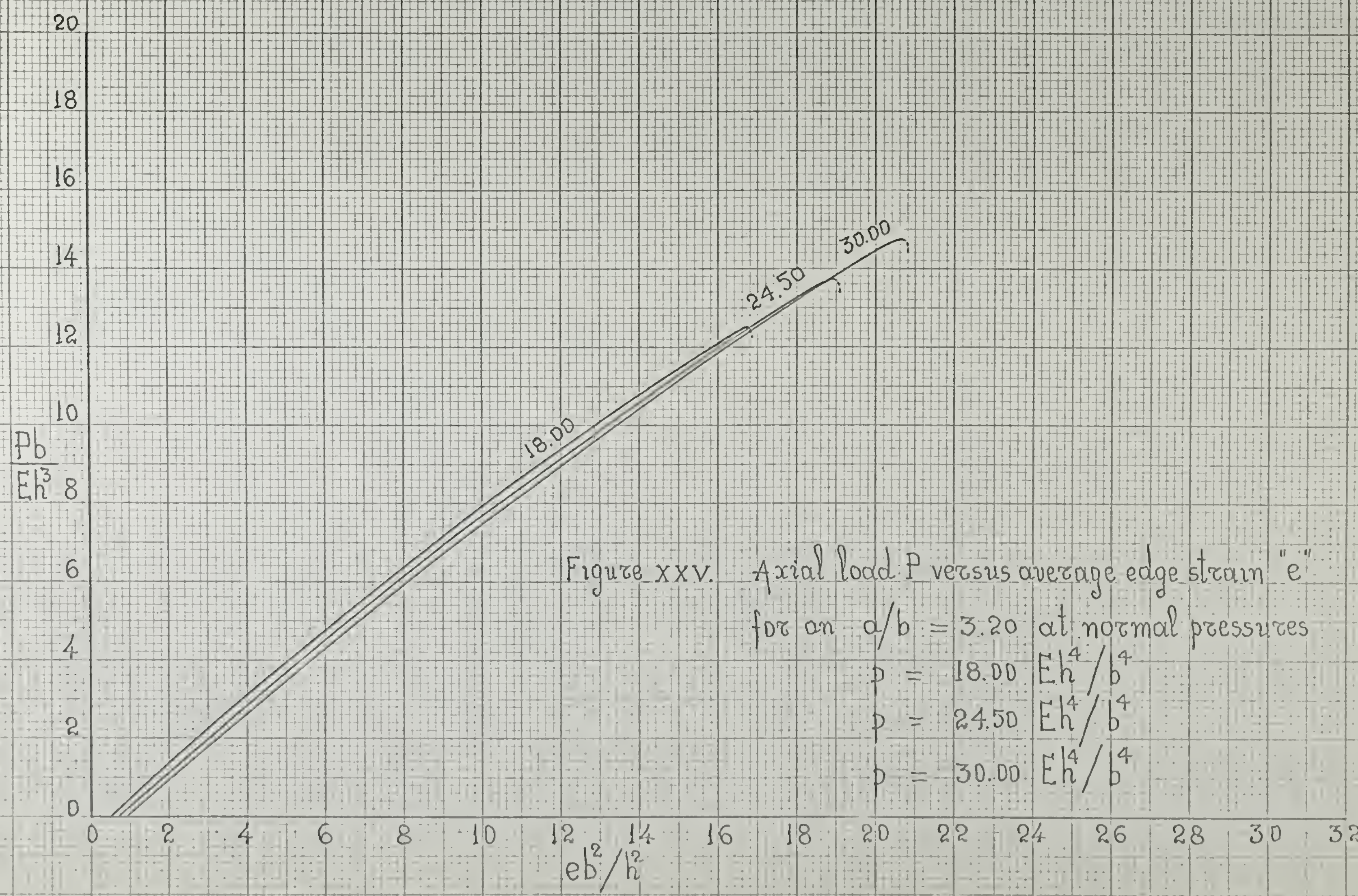

Figure XXV. Axial load P versus average edge strain "e"
for an $a/b = 3.20$ at normal pressures
$p = 18.00\ Eh^4/b^4$
$p = 24.50\ Eh^4/b^4$
$p = 30.00\ Eh^4/b^4$

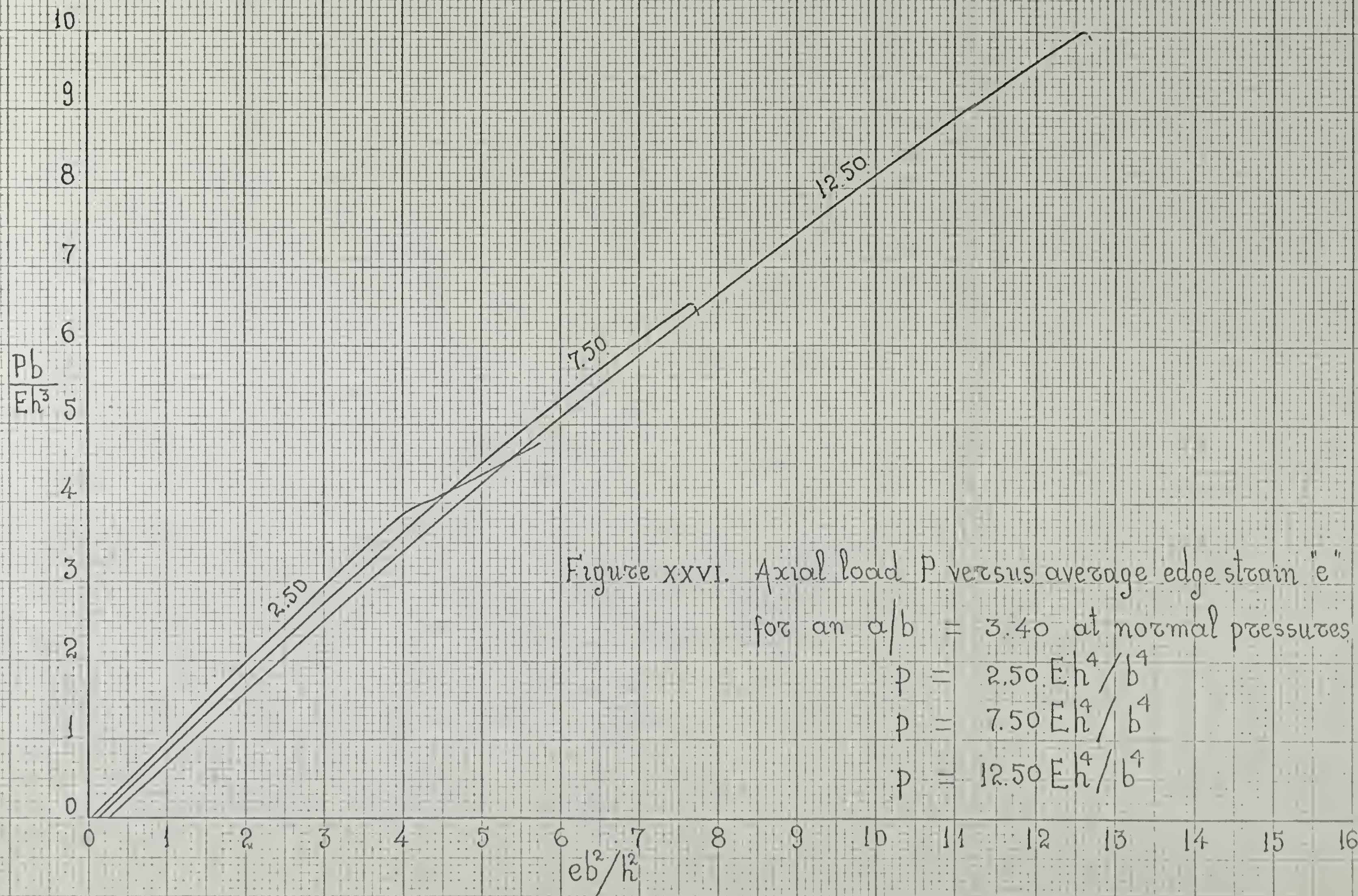

Figure XXVI. Axial load P versus average edge strain "e" for an $a/b = 3.40$ at normal pressures

$p = 2.50\, Eh^4/b^4$

$p = 7.50\, Eh^4/b^4$

$p = 12.50\, Eh^4/b^4$

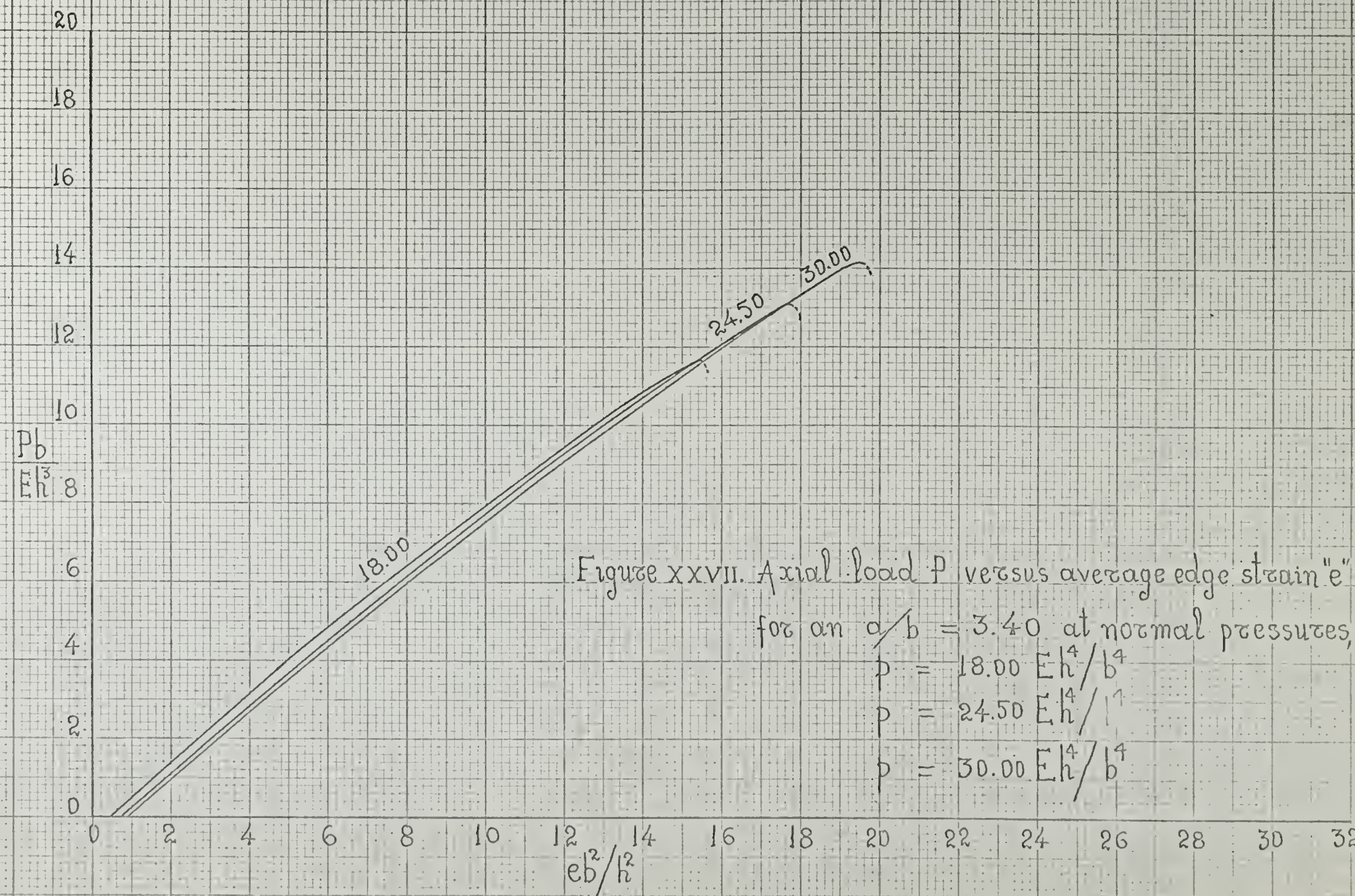

Figure XXVII. Axial load P versus average edge strain "e" for an $a/b = 3.40$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

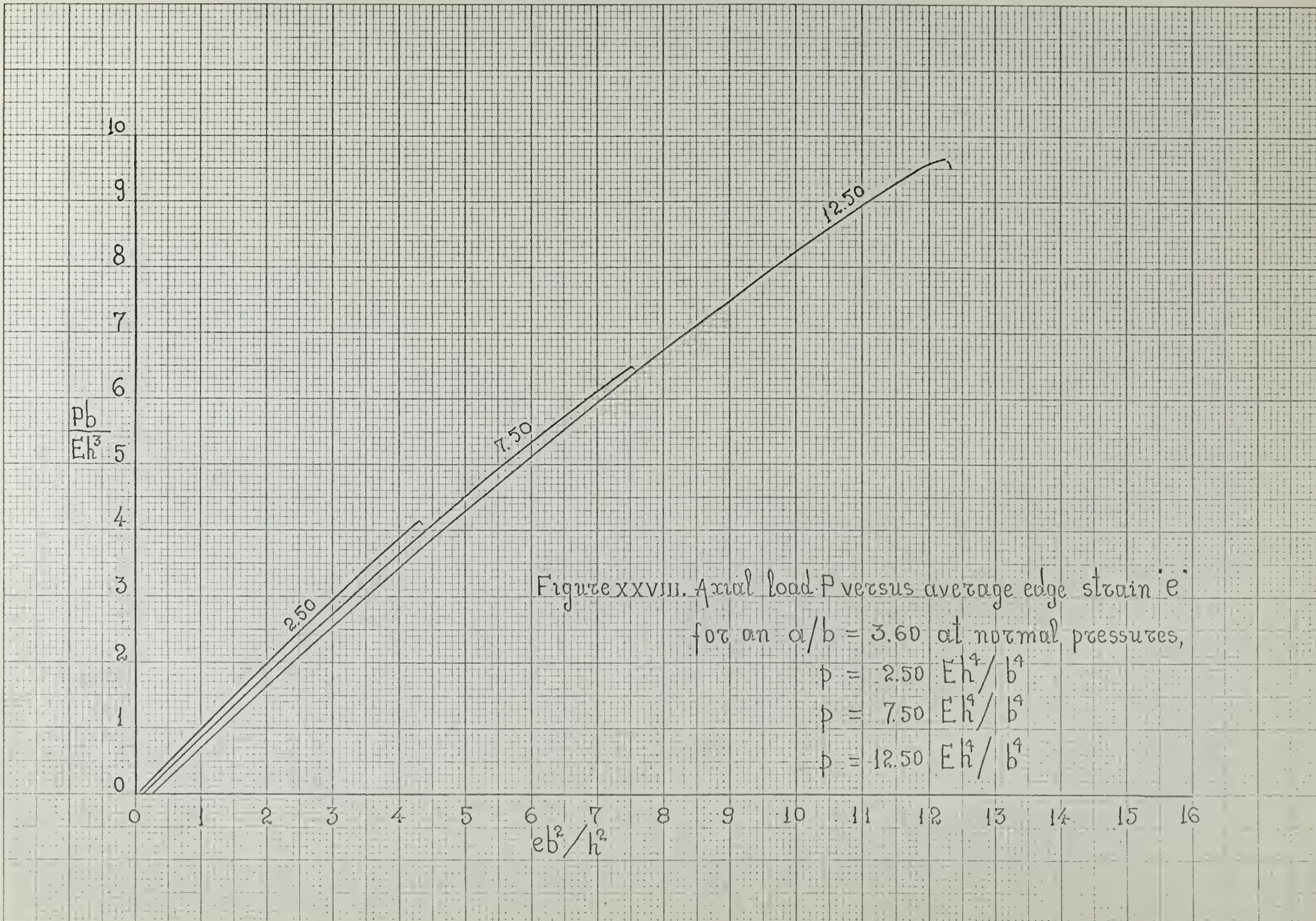

Figure XXVIII. Axial load P versus average edge strain "e" for an $a/b = 3.60$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

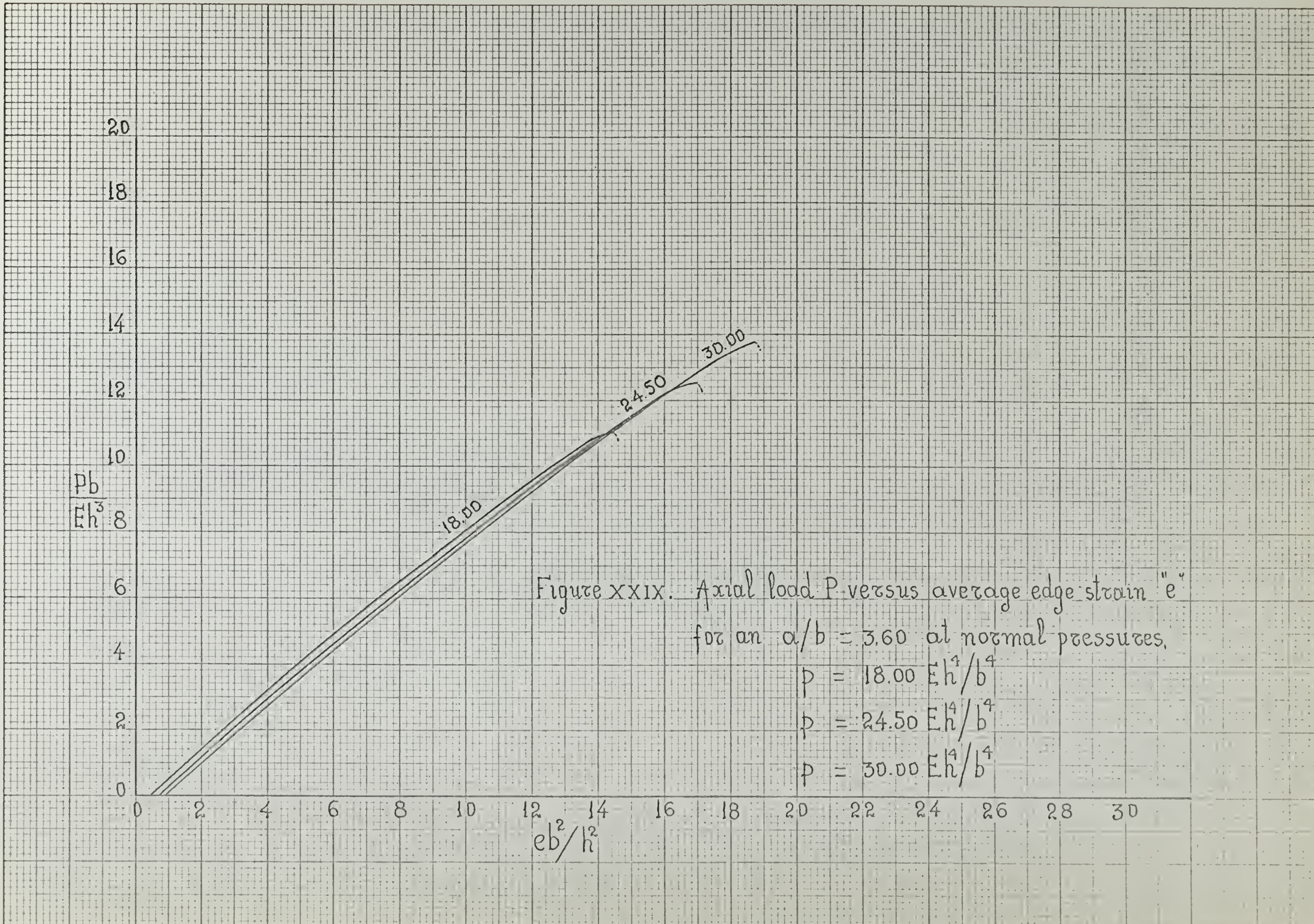

Figure XXIX. Axial load P versus average edge strain "e" for an $a/b = 3.60$ at normal pressures.

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

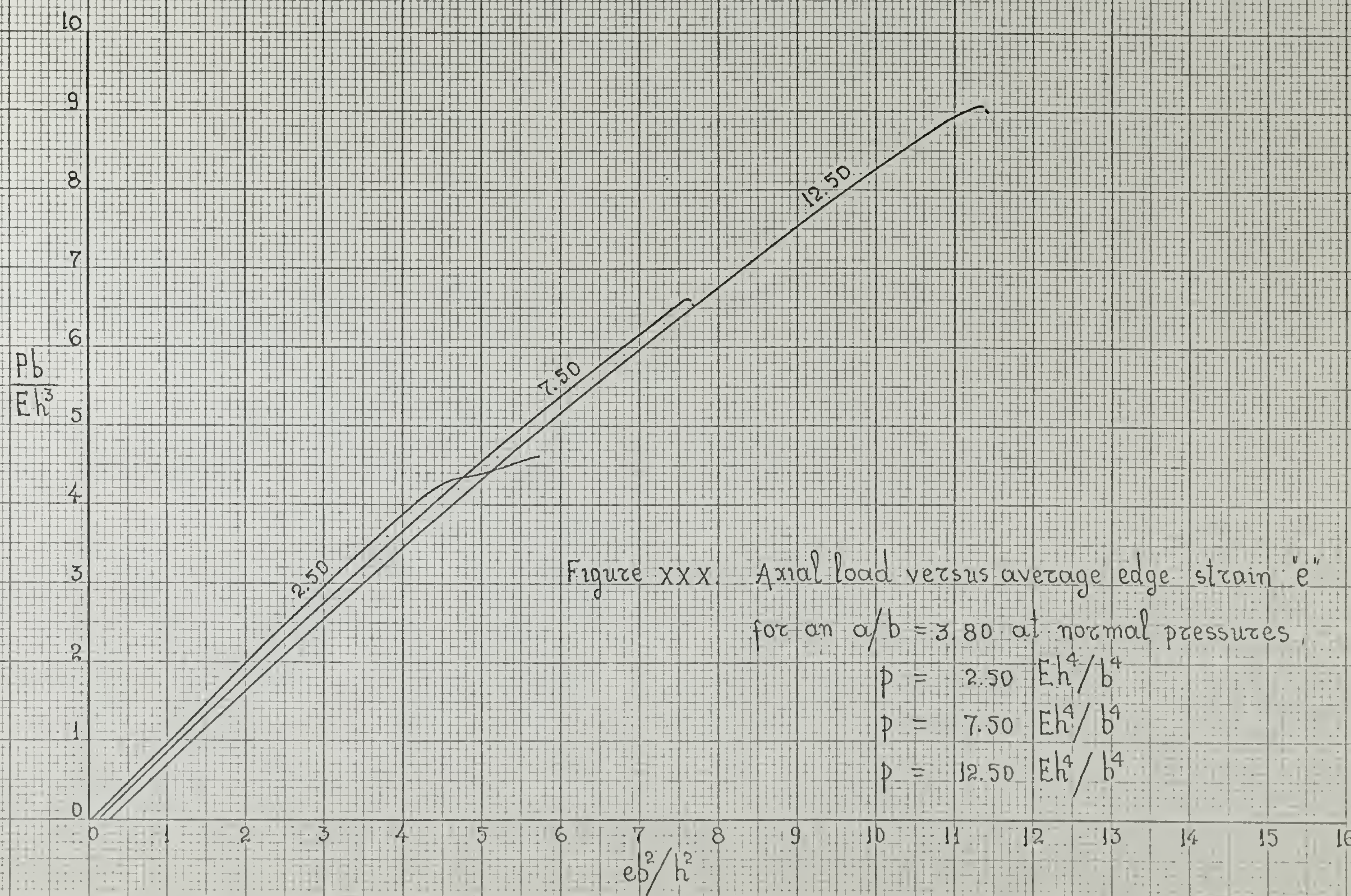

Figure XXX. Axial load versus average edge strain "e" for an $a/b = 3.80$ at normal pressures,

$p = 2.50\ Eh^4/b^4$

$p = 7.50\ Eh^4/b^4$

$p = 12.50\ Eh^4/b^4$

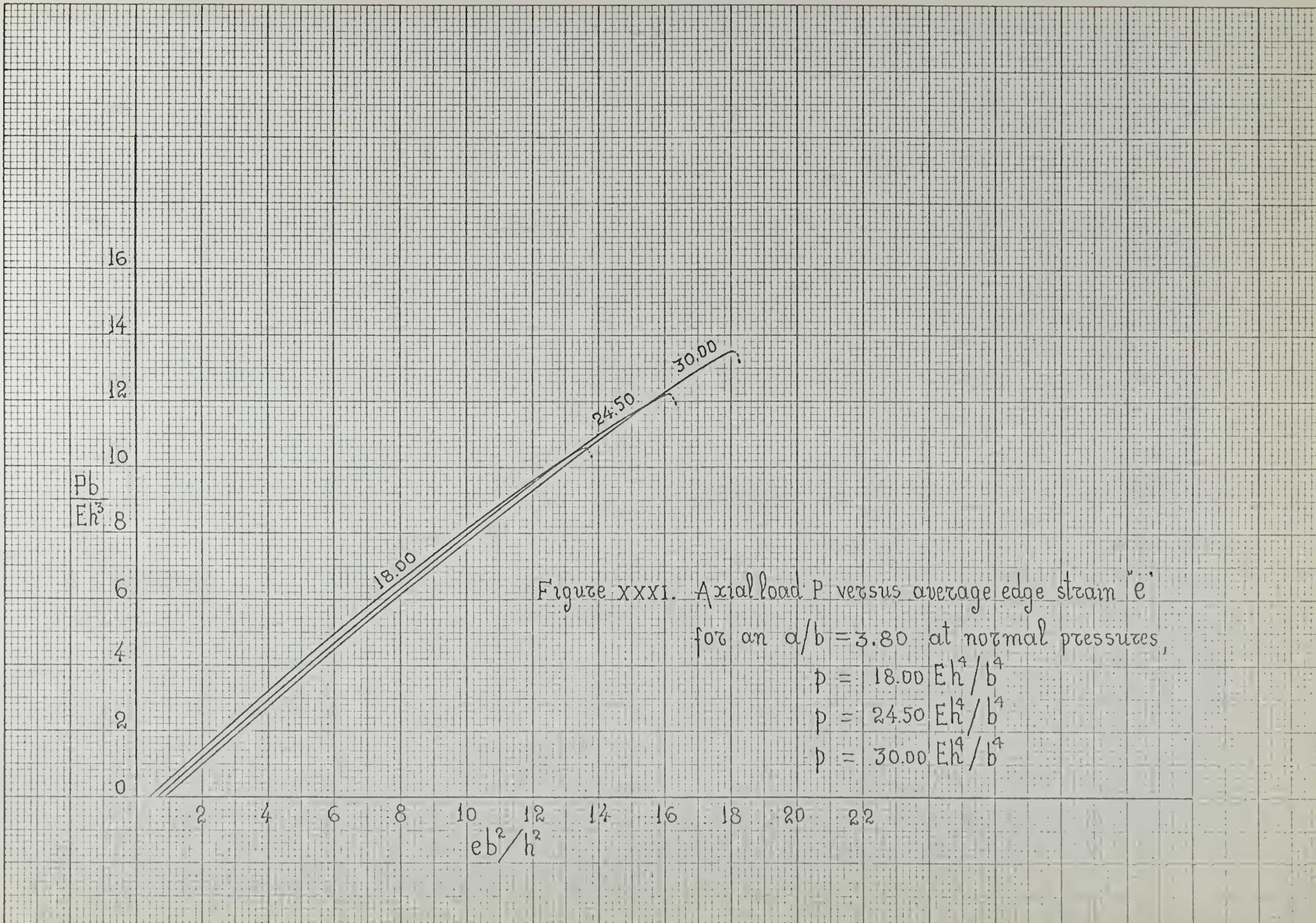

Figure XXXI. Axial load P versus average edge strain "e" for an $a/b = 3.80$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

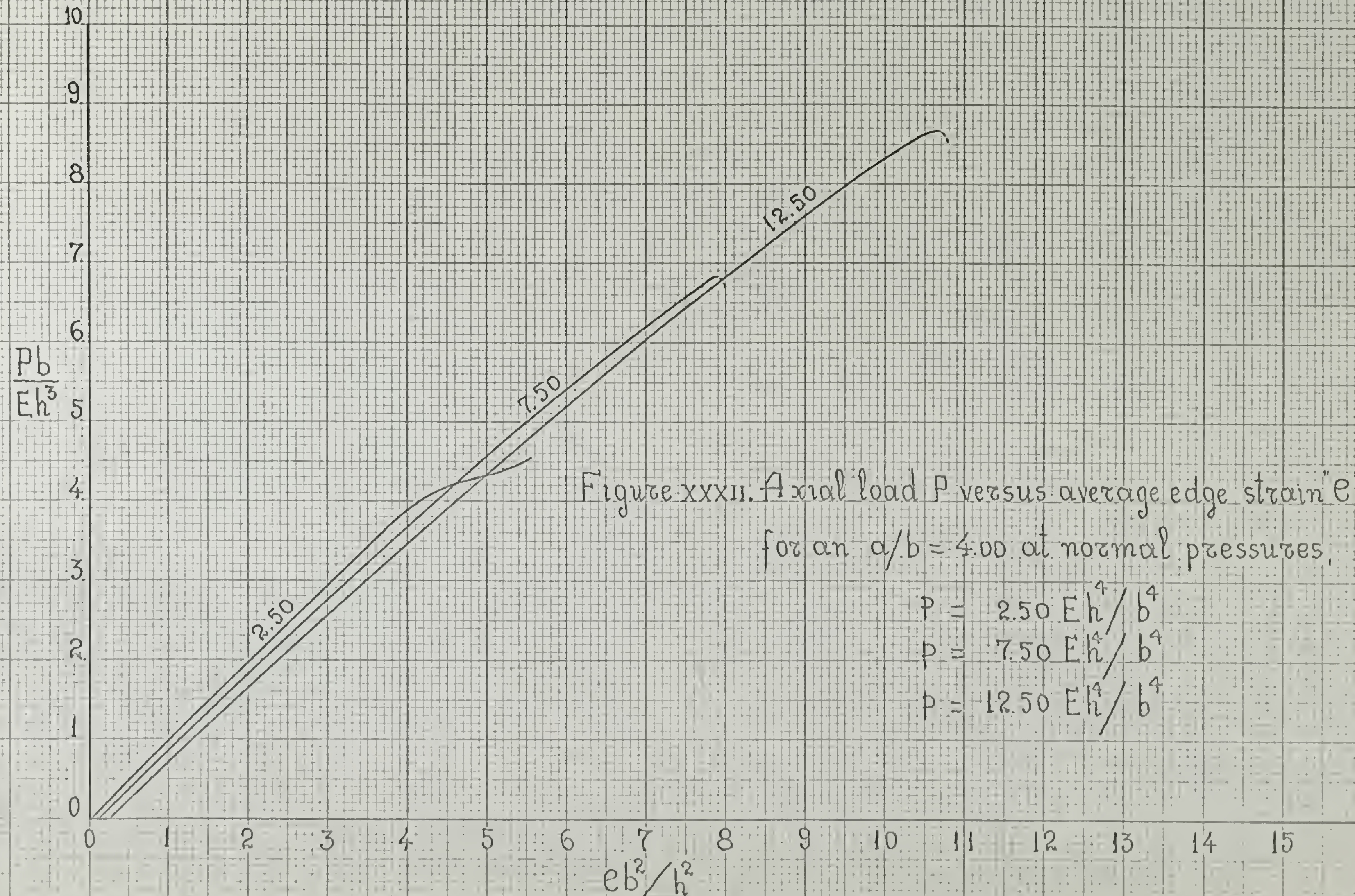

Figure XXXII. Axial load P versus average edge strain "e" for an $a/b = 4.00$ at normal pressures.

$P = 2.50\ Eh^4/b^4$

$P = 7.50\ Eh^4/b^4$

$P = 12.50\ Eh^4/b^4$

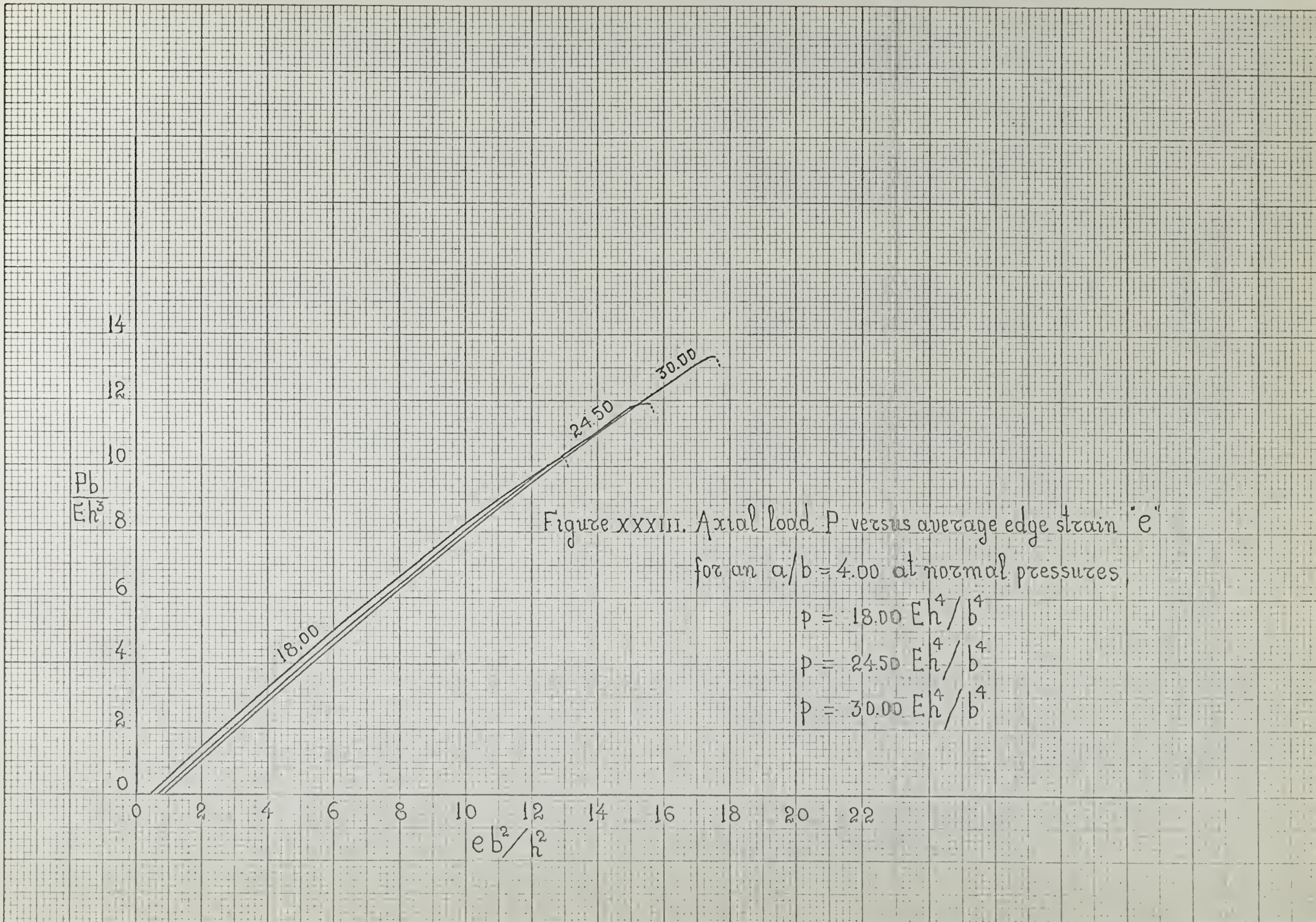

Figure XXXIII. Axial load P versus average edge strain "e" for an $a/b = 4.00$ at normal pressures,

$p = 18.00\ Eh^4/b^4$

$p = 24.50\ Eh^4/b^4$

$p = 30.00\ Eh^4/b^4$

Table XIII - Values of (Pb/Eh^3) critical at various values of a/b and pb^4/Eh^4. These are results using equation (8).

a/b	pb^4/Eh^4 =					
	2.50	7.50	12.50	18.00	24.50	30.00
0.60	79.51	79.60	79.68	79.78	79.89	79.98
1.00	27.56	28.07	28.59	29.17	29.86	30.46
1.20	16.89	17.49	18.06	18.66	19.32	19.86
1.40	11.33	12.29	13.13	13.95	14.84	15.54
1.60	7.90	9.35	10.48	11.57	12.74	13.67
1.70	6.77	8.41	9.68	10.93	12.30	13.43
1.80	5.93	7.71	9.14	10.60	12.31	13.86
1.90	5.36	7.19	8.82	10.62	13.20	19.35
2.00	4.96	6.83	8.72	11.38	17.03	17.65
2.10	4.69	6.54	8.96	14.79	15.68	16.38

Table XIV - Values of $(Pb/Eh^3)_{critical}$ at various values of a/b and pb^4/Eh^4. These are results using equation (7).

a/b	$pb^4/Eh^4 =$					
	2.50	7.50	12.50	18.00	24.50	30.00
4.00	4.20	6.82	8.68	10.26	11.98	13.34
3.95	4.16	6.78	8.77	10.33	12.03	13.37
3.90	4.16	6.72	8.87	10.41	12.08	13.40
3.85	4.20	6.67	8.97	10.50	12.14	13.45
3.80	4.30	6.61	9.08	10.58	12.21	13.50
3.60	4.12	6.49	9.64	11.04	12.57	13.77
3.50	4.03	6.49	10.00	11.32	12.80	13.96
3.40	3.98	6.53	10.00	11.65	13.06	14.19
3.20	3.92	6.71	9.13	12.50	13.74	14.77
3.10	3.92	6.86	8.98	13.09	14.17	15.13
3.08	3.92	6.89	8.97	12.17	14.26	15.21
3.04	3.92	6.97	8.96	11.69	14.47	15.38
3.00	3.91	7.05	8.96	11.42	14.70	15.57
2.95	3.91	7.17	8.98	11.19	15.03	15.82
2.90	3.92	7.30	9.03	11.05	13.98	16.11
2.85	3.94	7.44	9.10	10.97	13.40	16.47
2.80	3.99	7.60	9.19	10.93	13.08	15.19
2.60	4.02	8.44	9.79	11.17	12.71	13.98
2.57	4.04	6.89	9.91	11.25	12.74	13.95
2.50	4.10	6.50	10.24	11.50	12.87	13.97
2.40	4.20	6.33	10.83	11.97	13.20	14.18
2.30	4.30	6.31	11.58	12.60	13.72	14.61
2.25	4.36	6.33	9.15	12.99	14.06	14.90
2.20	4.48	6.39	8.81	13.44	14.45	15.26
2.15	4.53	6.47	8.64	11.64	14.91	15.67
2.10	4.69	6.57	8.56	10.93	15.43	16.16
2.05	4.82	6.70	8.56	10.61	13.12	--
2.00	4.97	6.86	8.61	10.45	12.55	14.30
1.90	--	7.27	8.69	10.45	12.15	13.48
1.70	6.87	8.68	10.06	11.36	12.71	13.74

V. DISCUSSION OF RESULTS

The most remarkable part of the results was the way the critical buckling load behaved with respect to varying values of a/b (Fig. XXXIV). As can be seen in the corrolary figure (page 64) to Fig. XXXIV, there is a striking resemblance to Bryan's buckling load solution of rectangular plates solely under the action of edge loadings. It may be recalled that the wave-like pattern of Bryan's curve of K versus a/b was due to the nature of the K-formula and to the minimizing procedure adopted in obtaining the critical load for any given value of a/b. Bryan's K-formula is a function of the plate's dimensions and of the number of half-sine waves the plate takes after buckling. Thus, it would seem from the foregoing statements to presuppose that the patterns obtained in this thesis are also due in part to the number of half-sine waves.

At first glance the results would give no indication as to the number of waves the plate takes at the incipience of buckling. But, checks had been made by the authors on the relationships between deflections at the midwidth of the plate and at various distances along its length. In other words plots were made of the deflection ratio, w/h, versus the distance ratio, x/a, at $y = b/2$. These were done for different (a/b)'s at the same normal pressure. For example, it was found that for a $p = 18.00\ Eh^4/b^4$ there were five buckles at $a/b = 2.40$. (The two half-sine waves resulting from the initial general downward deflection of the plate due to normal pressure were not included in the count.) When these checks were expanded to a wide range of values of a/b, it was found that points lying on the same trough of a p-curve gave the same number of buckles.

Since only representative tables are given (Tables I to XII) it was deemed unnecessary to insert all these checks in this thesis. If verifications are to be made on this particular point the authors suggest going through the programs shown in Appendix B.

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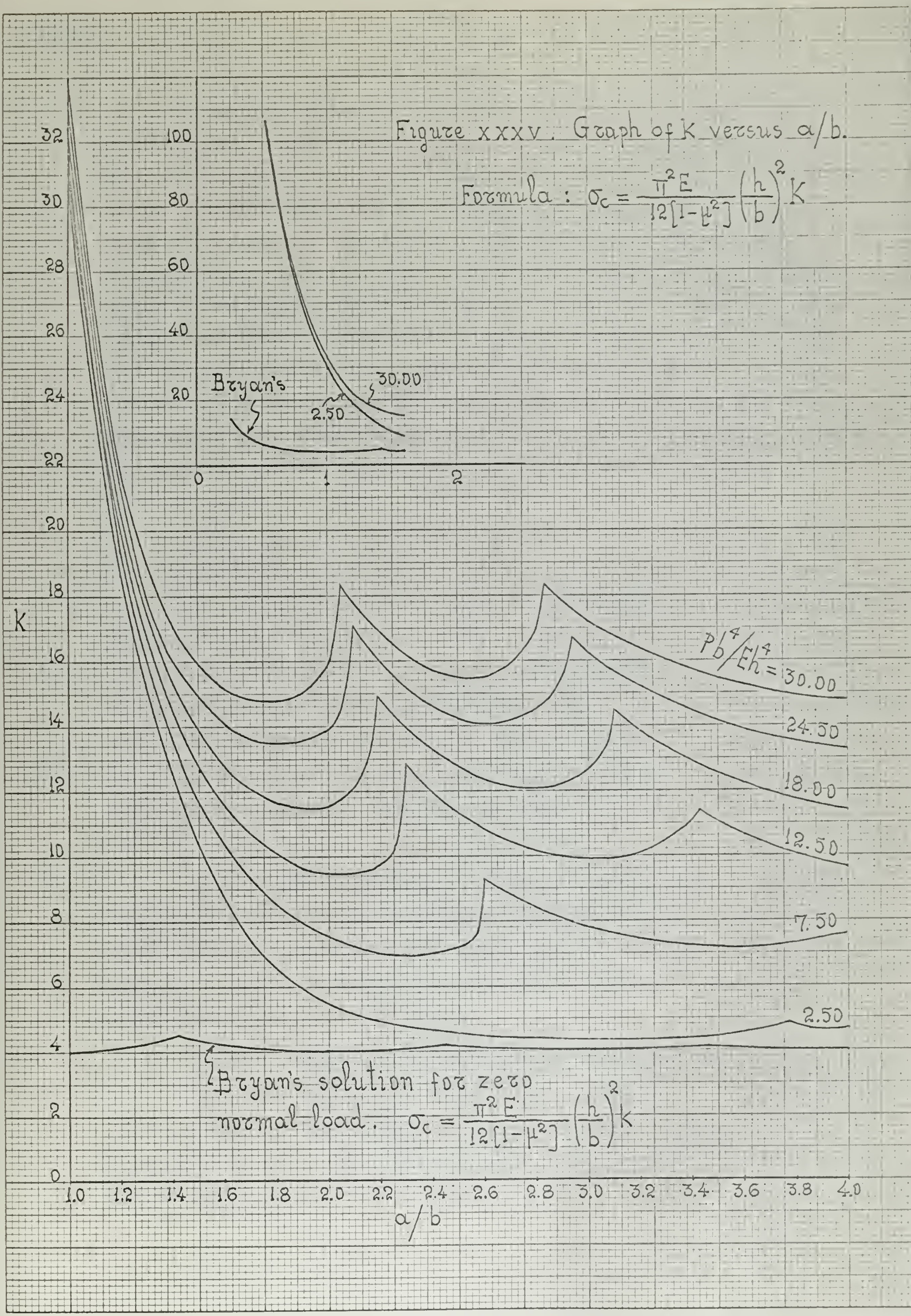
Figure XXXV. Graph of K versus a/b.
Formula : $\sigma_c = \frac{\pi^2 E}{12[1-\mu^2]}\left(\frac{h}{b}\right)^2 K$
Bryan's
30.00
2.50
$Pb^4/Eh^4 = 30.00$
24.50
18.00
12.50
7.50
2.50
Bryan's solution for zero normal load. $\sigma_c = \frac{\pi^2 E}{12[1-\mu^2]}\left(\frac{h}{b}\right)^2 K$
K
a/b

There is, however, no positive explanation as to why only an odd number of buckles is possible. There are either one, three, five, etc., buckles. An even number of buckles, which are obtainable in the case of Bryan's, seems entirely out of the question. On this particular aspect, a possible explanation that the authors believed could have happened was in the manner the deflection equations were obtained. The approximated deflection equations were follow-ups of those used in the references [2], [3], [4]. The equations only have the odd-numbered, subscripted deflection coefficients which, in the final analysis, resulted in an odd function for the w-equation. At this point, there is no way of predicting what the results will be if the even-numbered subscripts of the deflection coefficients are also taken into consideration.

In reference to the reliability of the results, it is to be noted that the basic theory is valid only within the range of Hooke's law. No allowance was made in the original differential equations on the change of Young's modulus when the axial load exceeds the proportional limit. Hence, all the results of this thesis presupposed that the material had been within the elastic region at all times. Experimental results are not available on the case of plates with the same loading and boundary conditions such that, definite identifications cannot be made as to whether the results are within the elastic region or not.

When a further study is made of Fig. XXXIV for a/b less than or equal to one, there is a very wide discrepancy between the calculated critical loads and Bryan's that it is quite doubtful whether the expected critical loads are below the yield strength of steel for a majority of plate characteristics.

The following computed results are hoped to illustrate the point:

Consider a plate with a $b/h = 104.7$ and acted on by a pressure, $p = 12.50\ Eh^4/b^4 = 3.11$ psi. The b/h may correspond to a 14.02-pound plate with $b = 36$ inches, and the p may correspond to a hydrostatic head of 7 feet.

There is, however, no positive explanation as to why only an odd number of buckles is possible. There are either one, three, five, etc., buckles. An even number of buckles, which are obtainable in the case of Bryan's, seems entirely out of the question. On this particular aspect, a possible explanation that the authors believed could have happened was in the manner the deflection equations were obtained. The approximated deflection equations were follow-ups of those used in the references [2], [3], [4]. The equations only have the odd-numbered, subscripted deflection coefficients which, in the final analysis, resulted in an odd function for the w-equation. At this point, there is no way of predicting what the results will be if the even-numbered subscripts of the deflection coefficients are also taken into consideration.

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The following computed results are hoped to illustrate the point:

Consider a plate with a $b/h = 105.7$ and acted on by a pressure, $p = 12.0\ Eh^4/b^4 = 2.11$ psi. The b/h may correspond to a 14.02-pound plate with $b = 36$ inches, and the p may correspond to a hydrostatic head of 7 feet.

Case 1	Case 2	Case 3
$a/b = 1.50$	$a/b = 1.00$	$a/b = 0.60$
$K \approx 12.8$	$K \approx 28$	$K = 78$
$\sigma_c = 31800$ psi	$\sigma_c = 69000$ psi	$\sigma_c = 192000$ psi

If mild steel or high-tensile steel (HTS) were used with yield stresses of 33000 and 47000 psi, respectively, it is apparent that cases 2 and 3 exceed their corresponding yield strengths. Hence, the theory fails for these cases. If HY80 or HY100 were used then theory is found to fail only for case 3. At any rate, the fact remains that the basic theory does not differentiate between the elastic and inelastic regions. A consequent conclusion, therefore, is the need for a check on the yield strength whenever the formulated σ_c of this thesis is used.

Except for the restrictions mentioned in the previous paragraph, it is obvious that the results obtained confirmed the conclusion of Levy on the effect of normal pressure on the buckling load. But, it is also evident that to neglect the normal pressure in predicting buckling strength is unwise design from an economic point of view.

Lastly, the computer programs are limited in scope, although they worked properly in providing all the needed results of this thesis. They were the consequent developments of the approximated deflections equations. A more desirable program would be to start with the equations shown in reference [2].

Case 1	Case 2	Case 3
$a/b = 1.50$	$a/b = 1.00$	$a/b = 0.50$
$K = 12.8$	$K = 29$	$K = 73$
$\sigma_c = 31800$ psi	$\sigma_c = 83000$ psi	$\sigma_c = 192000$ psi

If mild steel or high-tensile steel (HTS) were used with yield stresses of 33000 and 47000 psi, respectively, it is apparent that cases 2 and 3 exceed their corresponding yield strengths. Hence, the theory fails for these cases. If HY80 or HY100 were used then theory is found to fail only for case 3. At any rate, the fact remains that the basic theory does not differentiate between the elastic and inelastic regions. A consequent conclusion, therefore, is the need for a check on the yield strength whenever the formulated σ_c of this thesis is used.

Except for the restrictions mentioned in the previous paragraph, it is obvious that the results obtained confirmed the conclusion of Levy on the effect of normal pressure on the buckling load. But, it is also evident that to neglect the normal pressure in predicting buckling strength is unwise design from an economic point of view.

Lastly, the computer programs are limited in scope, although they worked properly in providing all the needed results of this thesis. They were the consequent developments of the approximated deflections equations. A more desirable program would be to start with the equations shown in reference [3].

VI. CONCLUSIONS AND RECOMMENDATIONS

Conclusions

1. Normal pressure always increases the buckling strength of rectangular flat plates. However, the buckling loads obtained by means of Levy's solution are not conclusive values; that is, the buckling load may or may not be within the proportional limit. In other words, theory does not differentiate between the elastic and inelastic regions.

2. Plotted values of critical loads against the ratio, a/b, for a constant value of normal pressure exhibits the same trend and characteristics as the plot of Bryan's formula for zero pressure.

3. To neglect the effect of normal pressure in the determination of buckling load is unwise design from an economic standpoint.

4. In general, the buckling load formula can be written in the form,

$$\sigma_c = \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{h}{b}\right)^2 K$$

where, this time, K is not only a function of plate geometry but also a function of the normal pressure and Poisson's ratio as well.

Recommendations

1. A better representative plot of K versus a/b is needed by including the even-numbered subscripted deflection coefficients of Levy's solution.

2. In consonance with the first recommendation, a revised computer program is desirable, possibly to start right at the series

forms of the equations shown in reference [2]. This step would eliminate using approximate deflection equations.

3. It is recommended that a study be made of the most feasible experimental set-up to simulate the boundary conditions stated in this thesis; and hence, the ensuing experimental verifications.

APPENDIX A
DEFLECTION EQUATIONS

The six (6) simultaneous cubic equations involved in the solution of the deflection equation,

$$w = w_{1,1} \sin\frac{\pi x}{a} \sin\frac{\pi y}{b} + w_{1,3} \sin\frac{\pi x}{a} \sin\frac{3\pi y}{b}$$

$$+ w_{3,1} \sin\frac{3\pi x}{a} \sin\frac{\pi y}{b} + w_{3,3} \sin\frac{3\pi x}{a} \sin\frac{3\pi y}{b}$$

$$+ w_{1,5} \sin\frac{\pi x}{a} \sin\frac{5\pi y}{b} + w_{5,1} \sin\frac{5\pi x}{a} \sin\frac{\pi y}{b}$$

are as follows*:

$$0 = -\frac{256\, a_1}{\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_3} w_{1,1} - \frac{16}{\pi^6} \frac{Pb}{Eh} w_{1,1}$$

$$+ (a_1 + a_2) w_{1,1}^3 - 3a_2 w_{1,1}^2 w_{1,3} - 3a_1 w_{1,1}^2 w_{3,1}$$

$$+ (9a_1 + 4a_2 + 16a_3 + a_8) w_{1,1} w_{1,3}^2 + (4a_1 + 9a_2 + 16a_3$$

$$+ a_4) w_{1,1} w_{3,1}^2 + 32a_3 w_{1,1} w_{1,3} w_{3,1} + 81(a_4 + a_8) w_{1,1} w_{3,3}^2$$

$$- 18(a_1 + a_8) w_{1,1} w_{1,3} w_{3,3} - 18(a_2 + a_4) w_{1,1} w_{3,1} w_{3,3}$$

$$- (9a_1 + 64a_3 + 25a_8) w_{1,3}^2 w_{3,1} - (9a_2 + 64a_3 + 25a_4) w_{1,3} w_{3,1}^2$$

$$+ (36a_1 + 36a_2 + 225a_4 + 225a_8) w_{1,3} w_{3,1} w_{3,3} - 6(a_2$$

$$+ 3a_8) w_{1,1} w_{1,3} w_{1,5} - 6(a_1 + 3a_4) w_{1,1} w_{3,1} w_{5,1}$$

$$+ 162a_8 w_{1,1} w_{3,3} w_{1,5} + 162a_4 w_{1,1} w_{3,3} w_{5,1} + (25a_1 + 4a_2$$

*The corresponding significance of each subscripted a is shown at the end of this Appendix.

APPENDIX A

DEFLECTION EQUATIONS

The six (6) simultaneous cubic equations involved in the solution of the deflection equation,

$$w = w_{1,1}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + w_{1,3}\sin\frac{\pi x}{a}\sin\frac{3\pi y}{b}$$

$$+ w_{3,1}\sin\frac{3\pi x}{a}\sin\frac{\pi y}{b} + w_{3,3}\sin\frac{3\pi x}{a}\sin\frac{3\pi y}{b}$$

$$+ w_{1,5}\sin\frac{\pi x}{a}\sin\frac{5\pi y}{b} + w_{5,1}\sin\frac{5\pi x}{a}\sin\frac{\pi y}{b}$$

are as follows*:

$$0 = -\frac{256\,a_1}{\pi^6}\,\frac{pb^4}{Eh^4} + \frac{4}{3(1-\nu^2)}\,\frac{h^2}{a^2}\,w_{1,1} - \frac{16}{\pi^6}\,\frac{P_0}{Eh}\,w_{1,1}$$

$$+ (a_1 + a_2)\,w_{1,1}^3 - 3a_2\,w_{1,1}^2\,w_{1,3} - 3a_1\,w_{1,1}^2\,w_{3,1}$$

$$+ (9a_1 + 4a_2 + 16a_3 + a_6)\,w_{1,1}\,w_{1,3}^2 + (4a_1 + 9a_2 + 16a_5$$

$$+ a_4)\,w_{1,1}\,w_{3,1}^2 + 32a_5\,w_{1,1}\,w_{1,3}\,w_{3,1} + 81(a_4 + a_8)\,w_{1,1}\,w$$

$$- 18(a_1 + a_8)\,w_{1,1}\,w_{1,3}\,w_{3,3} - 18(a_2 + a_4)\,w_{1,1}\,w_{3,1}\,w_{3,3}$$

$$- (9a_1 + 54a_3 + 25a_6)\,w_{1,3}^2\,w_{3,1} - (9a_2 + 64a_3 + 25a_4)\,w_{1,3}\,w$$

$$+ (25a_1 + 36a_2 + 225a_4 + 225a_8)\,w_{1,3}\,w_{3,1}\,w_{3,3} - 6(a_2$$

$$+ 3a_8)\,w_{1,1}\,w_{1,3}\,w_{1,5} - 6(a_1 + 3a_4)\,w_{1,1}\,w_{3,1}\,w_{5,1}$$

$$+ 162a_8\,w_{1,1}\,w_{3,3}\,w_{1,5} + 162a_4\,w_{1,1}\,w_{3,3}\,w_{5,1} + (25a_1 + 4a$$

*The corresponding significance of each subscripted a is shown at the end of this Appendix.

$$+ 81 a_8 + 16 a_9) w_{1,1} w_{1,5}^2 + (3a_2 + 64a_3) w_{1,3}^2 w_{1,5}$$

$$+ (3a_1 + 64a_3) w_{3,1}^2 w_{5,1} + (64a_3 + 274a_8) w_{1,3} w_{3,1} w_{1,5}$$

$$+ (64a_3 + 274a_4) w_{1,3} w_{3,1} w_{5,1} - 144 (a_3 + a_9) w_{1,3} w_{3,3} w_{1,5}$$

$$- 9(a_1 + 16a_3 + 49a_4 + 9a_8) w_{1,3} w_{3,3} w_{5,1} - 9(a_2 + 16a_3$$

$$+ 9a_4 + 49a_8) w_{3,1} w_{3,3} w_{1,5} - 144(a_3 + a_5) w_{3,1} w_{3,3} w_{5,1}$$

$$+ 9(a_2 + 81a_4) w_{3,3}^2 w_{1,5} + 9(a_1 + 81a_8) w_{3,3}^2 w_{5,1}$$

$$- (25a_2 + 441a_4 + 256a_5) w_{1,3} w_{5,1}^2 - (25a_1 + 441a_8$$

$$+ 256a_9) w_{3,1} w_{1,5}^2 + 729(a_4 + a_8) w_{3,3} w_{1,5} w_{5,1}.$$

$$0 = -\frac{256a_1}{3\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_9} w_{1,3} - \frac{16}{\pi^2} \frac{Pb}{Eh} w_{1,3} - a_2 w_{1,1}^3$$

$$+ (9a_1 + 4a_2 + 16a_3 + a_8) w_{1,1}^2 w_{1,3} + 16a_3 w_{1,1}^2 w_{3,1}$$

$$- 9(a_1 + a_8) w_{1,1}^2 w_{3,3} - (9a_2 + 64a_3 + 25a_4) w_{1,1} w_{3,1}^2$$

$$- 2(9a_1 + 64a_3 + 25a_8) w_{1,1} w_{1,3} w_{3,1} + (36a_1 + 36a_2$$

$$+ 225a_4 + 225a_8) w_{1,1} w_{3,1} w_{3,3} + (81a_1 + a_2) w_{1,3}^3$$

$$+ (272a_3 + 625a_4 + 625a_8) w_{1,3} w_{3,1}^2 - 243a_1 w_{1,3}^2 w_{3,3}$$

$$+ 9(36a_1 + a_2 + 144a_9 + 9a_{13}) w_{1,3} w_{3,3}^2 - 3(a_2 + 3a_8) w_{1,1}^2 w_{1,5}$$

$$+ 2(3a_2 + 64a_3) w_{1,1} w_{1,3} w_{1,5} + (64a_3 + 274a_8) w_{1,1} w_{3,1} w_{1,5}$$

$$+ (64a_3 + 274a_4) w_{1,1} w_{3,1} w_{5,1} - 144(a_3 + a_9) w_{1,1} w_{3,3} w_{1,5}$$

$$- 9(a_1 + 16a_3 + 49a_4 + 9a_8) w_{1,1} w_{3,3} w_{5,1} - (25a_2 + 441a_4$$

$$+ 256a_5) w_{1,1} w_{5,1}^2 - (9a_2 + 64a_3 + 1225a_8) w_{3,1}^2 w_{1,5}$$

$$- 256a_3 w_{3,1}^2 w_{5,1} - 512a_3 w_{1,3} w_{3,1} w_{1,5} - 2(9a_1 + 64a_3$$

$$+ 1225a_4) w_{1,3} w_{3,1} w_{5,1} + 9(2a_2 + 64a_3 + 225a_4$$

$$+ 256a_9 + 49a_{13}) w_{3,1} w_{3,3} w_{1,5} + 9(2a_1 + 64a_3 + 256a_5$$

$$+ 225a_8 + 49a_{12}) w_{3,1} w_{3,3} w_{5,1} + (225a_1 + 4a_2 + 256a_3$$

$$+ a_{10}) w_{1,3} w_{1,5}^2 + (256a_3 + 2401a_4 + 4096a_5$$

$$+ 2401a_{12}) w_{1,3} w_{5,1}^2 + 656a_3 w_{3,1} w_{1,5} w_{5,1} - 9(25a_1$$

$$+ 64a_3 + 9a_{10}) w_{3,3} w_{1,5}^2 - 9(64a_3 + 144a_4 + 169a_{13}) w_{3,3} w_{1,5} w_{5,1}$$

$$- (25a_2 + 576a_3 + 8281a_{12}) w_{1,5} w_{5,1}^2 .$$

$$0 = - \frac{256a_1}{3\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_5} w_{3,1} - \frac{144}{\pi^2} \frac{Pb}{Eh} w_{3,1} - a_1 w_{1,1}^3$$

$$+ 16a_3 w_{1,1}^2 w_{1,3} + (4a_1 + 9a_2 + 16a_3 + a_4) w_{1,1}^2 w_{3,1} - 9(a_2$$

$$+ a_4) w_{1,1}^2 w_{3,3} - (9a_1 + 64a_3 + 25a_8) w_{1,1} w_{1,3}^2 - 2(9a_2$$

$$+ 64a_3 + 25a_4) w_{1,1} w_{1,3} w_{3,1} + (36a_1 + 36a_2 + 225a_4$$

$$+ 225a_8) w_{1,1} w_{1,3} w_{3,3} + (272a_3 + 625a_4 + 625a_8) w_{1,3}^2 w_{3,1}$$

$$+ (a_1 + 81a_2) w_{3,1}^3 - 243a_2 w_{3,1}^2 w_{3,3} + 9(a_1 + 36a_2 + 144a_5$$

$$+ 9a_{12}) w_{3,1} w_{3,3}^2 - 3(a_1 + 3a_4) w_{1,1}^2 w_{5,1} + (64a_3$$

$$+ 274a_8) w_{1,1} w_{1,3} w_{1,5} + (64a_3 + 274a_4) w_{1,1} w_{1,3} w_{5,1}$$

$$+ 2(3a_1 + 64a_3) w_{1,1} w_{3,1} w_{5,1} - 9(2a_2 + 16a_3 + 9a_4$$

$$+ 49a_8) w_{1,1} w_{3,3} w_{1,5} - 144(a_3 + a_5) w_{1,1} w_{3,3} w_{5,1}$$

$$- (25a_1 + 441a_8 + 256a_9) w_{1,1} w_{1,5}^2 - 256a_3 w_{1,3}^2 w_{1,5}$$

$$- (9a_1 + 64a_3 + 1225a_4) w_{1,3}^2 w_{5,1} - 2(9a_2 + 64a_3$$

$$+ 1225a_8) w_{1,3} w_{3,1} w_{1,5} - 512a_3 w_{1,3} w_{3,1} w_{5,1} + 9(2a_2$$

$$+ 64a_3 + 225a_4 + 256a_9 + 49a_{13}) w_{1,3} w_{3,3} w_{1,5}$$

$$+ 9(2a_1 + 64a_3 + 256a_5 + 225a_8 + 49a_{12}) w_{1,3} w_{3,3} w_{5,1}$$

$$+ (256a_3 + 2401a_8 + 4096a_9 + 2401a_{13}) w_{3,1} w_{1,5}^2$$

$$+ (4a_1 + 225a_2 + 256a_3 + a_6) w_{3,1} w_{5,1}^2 + 656a_3 w_{1,3} w_{1,5} w_{5,1}$$

$$- 9(25a_2 + 64a_3 + 9a_6) w_{3,3} w_{5,1}^2 - 9(64a_3 + 441a_8$$

$$+ 169a_{12}) w_{3,3} w_{1,5} w_{5,1} - (25a_1 + 576a_3 + 8281a_{13}) w_{1,5}^2 w_{5,1}.$$

$$0 = - \frac{256a_1}{9\pi^6} \frac{pb^4}{Eh} + \frac{324}{3(1-\mu^2)} \frac{h^2}{a_3} w_{3,3} - \frac{144}{\pi^2} \frac{Pb}{Eh} w_{3,3}$$

$$- 9(a_1 + a_8) w_{1,1}^2 w_{1,3} - 9(a_2 + a_4) w_{1,1}^2 w_{3,1} + 81(a_4 + a_8) w_{1,1}^2 w_{3,3}$$

$$+ (36a_1 + 36a_2 + 225a_4 + 225a_8) w_{1,1} w_{1,3} w_{3,1} - 81a_1 w_{1,3}^3$$

$$- 81a_2 w_{3,1}^3 + 9(36a_1 + a_2 + 144a_9 + 9a_{13}) w_{1,3}^2 w_{3,3}$$

$$+ 81(a_1 + a_2) w_{3,3}^3 + 9(a_1 + 36a_2 + 144a_5 + 9a_{12}) w_{3,1}^2 w_{3,3}$$

$$+ 81a_8 w_{1,1}^2 w_{1,5} + 81a_4 w_{1,1}^2 w_{5,1} - 144(a_3 + a_9) w_{1,1} w_{1,3} w_{1,5}$$

$$- 9(2a_1 + 16a_3 + 49a_4 + 9a_8) w_{1,1} w_{1,3} w_{5,1} - 9(2a_2$$

$$+ 16a_3 + 9a_4 + 49a_8) w_{1,1} w_{3,1} w_{1,5} - 144(a_3 + a_5) w_{1,1} w_{3,1} w_{5,1}$$

$$+ 18(a_2 + 81a_4) w_{1,1} w_{3,3} w_{1,5} + 18(a_1 + 18a_8) w_{1,1} w_{3,3} w_{5,1}$$

$$+ 729(a_4 + a_8) w_{1,1} w_{1,5} w_{5,1} + 9(2a_2 + 64a_3 + 225a_4$$

$$+ 256a_9 + 49a_{13}) w_{1,3} w_{3,1} w_{1,5} + 9(2a_1 + 64a_3 + 256a_5$$

$$+ 225a_8 + 49a_{12}) w_{1,3} w_{3,1} w_{5,1} - 9(25a_1 + 64a_3$$

$$+ 9a_{10}) w_{1,3} w_{1,5}^2 - 9(25a_2 + 64a_3 + 9a_6) w_{3,1} w_{5,1}^2$$

$$- 9(64a_3 + 441a_4 + 169a_{13}) w_{1,3} w_{1,5} w_{5,1} - 9(64a_3$$

$$+ 441a_8 + 169a_{12}) w_{3,1} w_{1,5} w_{5,1} + 81(16a_3 + 81a_4$$

$$+ a_8 + 81a_{10}) w_{3,3} w_{1,5}^2 + 81(16a_3 + a_4 + 81a_6$$

$$+ 81\, a_8) w_{3,3} w_{5,1}^2 + 2592 a_3 w_{3,3} w_{5,1} \, .$$

$$0 = - \frac{256 a_1}{5\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_{11}} w_{1,5} - \frac{16}{\pi^2} \frac{Pb}{Eh} w_{1,5}$$

$$- 3(a_2 + 3a_8) w_{1,1}^2 w_{1,3} + 81 a_8 + (3a_2 + 64 a_3) w_{1,1} w_{1,3}^2$$

$$+ (64 a_3 + 274 a_8) w_{1,1} w_{1,3} w_{3,1} + 9(a_2 + 81 a_4) w_{1,1} w_{3,3}^2$$

$$- 144(a_3 + a_9) w_{1,1} w_{1,3} w_{3,3} - 9(2a_2 + 16 a_3 + 9 a_4$$

$$+ 49 a_8) w_{1,1} w_{3,1} w_{3,3} - 256 a_3 w_{1,3}^2 w_{3,1} - (9a_2 + 64 a_3$$

$$+ 1225 a_8) w_{1,3} w_{3,1}^2 + 9(2a_2 + 64 a_3 + 225 a_4 + 256 a_9$$

$$+ 49 a_{13}) w_{1,3} w_{3,1} w_{3,3} + (25 a_1 + 4 a_2 + 81 a_8 + 16 a_9) w_{1,1}^2 w_{1,5}$$

$$- 2(25 a_1 + 441 a_8 + 256 a_9) w_{1,1} w_{3,1} w_{1,5} + 729(a_4$$

$$+ a_8) w_{1,1} w_{3,3} w_{5,1} + (225 a_1 + 4 a_2 + 256 a_3 + a_{10}) w_{1,3}^2 w_{1,5}$$

$$+ (256 a_3 + 2401 a_8 + 4096 a_9 + 2401 a_{13}) w_{3,1}^2 w_{1,5}$$

$$+ 656 a_3 w_{1,3} w_{3,1} w_{5,1} - 18(25 a_1 + 64 a_3 + 9 a_{10}) w_{1,3} w_{3,3} w_{1,5}$$

$$- 9(64 a_3 + 441 a_4 + 169 a_{13}) w_{1,3} w_{3,3} w_{5,1} - 9(64 a_3 + 441 a_8$$

$$+ 169 a_{12}) w_{3,1} w_{3,3} w_{5,1} + 81(16 a_3 + 81 a_4 + a_8 + 81 a_{10}) w_{3,3}^2 w_{1,5}$$

$$+ 1296 a_3 w_{3,3}^2 w_{5,1} - (25 a_2 + 576 a_3 + 8281 a_{12}) w_{1,3} w_{5,1}^2$$

$$- 2(25 a_1 + 576 a_3 + 8281 a_{13}) w_{3,1} w_{1,5} w_{5,1} + (625 a_1 + a_2) w_{1,5}^3$$

$$+ (1552 a_3 + 28561 a_{12} + 28561 a_{13}) w_{1,5} w_{5,1}^2 \, .$$

$$0 = - \frac{256 a_1}{5\pi^6} \frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)} \frac{h^2}{a_7} w_{5,1} - \frac{400}{\pi^2} \frac{Pb}{Eh} w_{5,1}$$

$$- 3(a_1 + 3a_4) w_{1,1}^2 w_{3,1} + 81 a_4 w_{1,1}^2 w_{3,3} + (3a_1 + 64 a_3) w_{1,1} w_{3,1}^2$$

$$+ (64a_3 + 274a_4) w_{1,1} w_{1,3} w_{3,1} + 9(a_1 + 81a_8) w_{1,1} w_{3,3}^2$$

$$- 9(2a_1 + 16a_3 + 49a_4 + 9a_8) w_{1,1} w_{1,3} w_{3,3} - 144(a_3$$

$$+ a_5) w_{1,1} w_{3,1} w_{3,3} - (9a_1 + 64a_3 + 1225a_4) w_{1,3}^2 w_{3,1}$$

$$- 256a_3 w_{1,3} w_{3,1}^2 + 9(2a_1 + 64a_3 + 256a_5 + 225a_8$$

$$+ 49a_{12}) w_{1,3} w_{3,1} w_{3,3} + (4a_1 + 25a_2 + 81a_4 + 16a_5) w_{1,1}^2 w_{5,1}$$

$$- 2(25a_2 + 441a_4 + 256a_5) w_{1,1} w_{1,3} w_{5,1} + 729(a_4$$

$$+ a_8) w_{1,1} w_{3,3} w_{1,5} + (256a_3 + 2401a_4 + 4096a_5 + 2401a_{12}) w_{1,3}^2 w_{5,1}$$

$$+ (4a_1 + 225a_2 + 256a_3 + a_6) w_{3,1}^2 w_{5,1} + 656a_3 w_{1,3} w_{3,1} w_{1,5}$$

$$- 9(64a_3 + 441a_4 + 169a_{13}) w_{1,3} w_{3,3} w_{1,5} - 9(64a_3 + 441a_8$$

$$+ 169a_{12}) w_{3,1} w_{3,3} w_{1,5} - 18(25a_2 + 64a_3 + 9a_6) w_{3,1} w_{3,3} w_{5,1}$$

$$+ 1296a_3 w_{3,3}^2 w_{1,5} + 81(16a_3 + a_4 + 81a_6 + 81a_8) w_{3,3}^2 w_{5,1}$$

$$- (25a_1 + 576a_3 + 8281a_{13}) w_{3,1} w_{1,5}^2 - 2(25a_2 + 576a_3$$

$$+ 8281a_{12}) w_{1,3} w_{1,5} w_{5,1} + (1552a_3 + 28561a_{12} + 28561a_{13}) w_{1,5}^2 w_{5,1}$$

$$+ (a_1 + 625a_2) w_{5,1}^3.$$

The four (4) simultaneous equations involved in the solution of the deflection equation,

$$w = w_{1,1} \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} + w_{3,1} \sin \frac{3\pi x}{a} \sin \frac{\pi y}{b}$$

$$+ w_{5,1} \sin \frac{5\pi x}{a} \sin \frac{\pi y}{b} + w_{7,1} \sin \frac{7\pi x}{a} \sin \frac{\pi y}{b},$$

are as follows:

$$0 = -\frac{256a_1}{\pi^6}\frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)}\frac{h^2}{a_3}w_{1,1} - \frac{16}{\pi^2}\frac{Pb}{Eh}w_{1,1} + (a_1+a_2)w_{1,1}^3$$

$$- 3a_1 w_{1,1}^2 w_{3,1} + (4a_1 + 9a_2 + 16a_3 + a_4) w_{1,1} w_{3,1}^2$$

$$+ (3a_1 + 64a_3) w_{3,1}^2 w_{5,1} - (3a_1 + 25a_4) w_{3,1}^2 w_{7,1} + (4a_1$$

$$+ 25a_2 + 81a_4 + 16a_5) w_{1,1} w_{5,1}^2 + (4a_1 + 49a_2 + 256a_5$$

$$+ 81a_6) w_{1,1} w_{7,1}^2 - 6(a_1 + 3a_4) w_{1,1} w_{3,1} w_{5,1} - 2(3a_1$$

$$+ 64a_5) w_{1,1} w_{5,1} w_{7,1} + (6a_1 + 144a_3 + 225a_4$$

$$+ 9a_6) w_{3,1} w_{5,1} w_{7,1}.$$

$$0 = -\frac{256a_1}{3\pi^6}\frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)}\frac{h^2}{a_5}w_{3,1} - \frac{144}{\pi^2}\frac{Pb}{Eh}w_{3,1} - a_1 w_{1,1}^3$$

$$+ (4a_1 + 9a_2 + 16a_3 + a_4) w_{1,1}^2 w_{3,1} - 3(a_1 + 3a_4) w_{1,1}^2 w_{5,1}$$

$$+ (a_1 + 81a_2) w_{3,1}^3 + (4a_1 + 225a_2 + 256a_3 + a_6) w_{3,1} w_{5,1}^2$$

$$+ 3(a_1 + 192a_3) w_{5,1}^2 w_{7,1} + (4a_1 + 441a_2 + 625a_4 + 16a_7) w_{3,1} w_{7,1}^2$$

$$+ 2(3a_1 + 64a_3) w_{1,1} w_{3,1} w_{5,1} - 2(3a_1 + 25a_4) w_{1,1} w_{3,1} w_{7,1}$$

$$+ 3(2a_1 + 48a_3 + 75a_4 + 3a_6) w_{1,1} w_{5,1} w_{7,1}.$$

$$0 = -\frac{256a_1}{5\pi^6}\frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)}\frac{h^2}{a_7}w_{5,1} - \frac{400}{\pi^2}\frac{Pb}{Eh}w_{5,1}$$

$$- 3(a_1 + 3a_4) w_{1,1}^2 w_{3,1} + (4a_1 + 25a_2 + 81a_4 + 16a_5) w_{1,1}^2 w_{5,1}$$

$$- (3a_1 + 64a_5) w_{1,1}^2 w_{7,1} + (3a_1 + 64a_3) w_{1,1} w_{3,1}^2 + (4a_1 + 225a_2$$

$$+ 256a_3 + a_6) w_{3,1}^2 w_{5,1} + (a_1 + 625a_2) w_{5,1}^3 + (4a_1 + 1225a_2$$

$$+ 1296a_3 + a_4) w_{5,1} w_{7,1}^2 + 3(2a_1 + 48a_3 + 75a_4 + 3a_6) w_{1,1} w_{3,1} w_{7,1}$$

$$+ 6(a_1 + 192a_3) w_{3,1} w_{5,1} w_{7,1}.$$

$$0 = -\frac{256a_1}{7\pi^6}\frac{pb^4}{Eh} + \frac{4}{3(1-\mu^2)}\frac{h^2}{a_{15}} w_{7,1} - \frac{784}{\pi^2}\frac{Pb}{Eh} w_{7,1}$$

$$- (3a_1 + 64a_5) w_{1,1}^2 w_{5,1} + (4a_1 + 49a_2 + 256a_5 + 81a_6) w_{3,1}^2 w_{7,1}$$

$$- (3a_1 + 25a_4) w_{1,1} w_{3,1}^2 + (4a_1 + 441a_2 + 625a_4 + 16a_7) w_{3,1}^2 w_{7,1}$$

$$+ 3(a_1 + 192a_3) w_{3,1} w_{5,1}^2 + (4a_1 + 1225a_2 + 1296a_3 + a_{14}) w_{5,1}^2 w_{7,1}$$

$$+ (a_1 + 2401a_2) w_{7,1}^3 + 3(2a_1 + 48a_3 + 75a_4 + 3a_6) w_{1,1} w_{3,1} w_{5,1}.$$

The subscripted a's have the following significance:

$a_1 = \alpha^2$

$a_2 = 1/\alpha^2$

$a_3 = \alpha^2/(\alpha^2 + 1)^2$

$a_4 = \alpha^2/(\alpha^2 + 4)^2$

$a_5 = \alpha^2/(\alpha^2 + 9)^2$

$a_6 = \alpha^2/(\alpha^2 + 16)^2$

$a_7 = \alpha^2/(\alpha^2 + 25)^2$

$a_8 = \alpha^2(4\alpha^2 + 1)^2$

$a_9 = \alpha^2/(9\alpha^2 + 1)^2$

$a_{10} = \alpha^2/(16\alpha^2 + 1)^2$

$a_{11} = \alpha^2/(25\alpha^2 + 1)^2$

$a_{12} = \alpha^2/(4\alpha^2 + 9)^2$

$a_{13} = \alpha^2/(9\alpha^2 + 4)^2$

$a_{14} = \alpha^2/(\alpha^2 + 36)^2$

$a_{15} = \alpha^2/\alpha^2 + 49)^2$

where $\alpha = a/b$.

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C
C                SOLUTION OF THE DEFLECTION EQUATION
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$$W = W_{1,1}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + W_{3,1}\sin\frac{3\pi x}{a}\sin\frac{\pi y}{b} + W_{5,1}\sin\frac{5\pi x}{a}\sin\frac{\pi y}{b} + W_{7,1}\sin\frac{7\pi x}{a}\sin\frac{\pi y}{b}$$

```
C                THIS METHOD IS SET UP FOR A CONTINUOUS
C            CHANGE IN AXIAL LOAD FROM ZERO TO A VALUE WITHIN
C            THE VICINITY OF THE CRITICAL BUCKLING LOAD.  ZERO
C            LATERAL LOAD IS EXCLUDED.
C
C                MAIN PROGRAM
C
*     XEQ
*     LABEL
*     LIST8
*     FORTRAN
      READ 1,  IND
    1 FORMAT (I5)
      PRINT 2
    2 FORMAT (82H1  PNORM   AXIAL     W(1,1)/H  W(3,1)/H  W(5,1)/H  W(7,
     11)/H  STRAIN         ERROR  )
      DO 44  JOB = 1,IND
      READ 3, U, ALPHA, PNORM, LOOP, LAP
    3 FORMAT (3F8.0,2I8)
      DIMENSION  B1(4), B2(4), B3(4)
      CALL CONST (U, ALPHA, PNORM, A1, A2, A3, A4, A5, A6, A7, A8, A9,
     1           A10, A11, A12, A13, A14, A15, A16, A17, B1, B2, B3)
C              INITIAL ESTIMATES OF W(I)'S
```

```
      DIMENSION W(4)
      W(1) = 0.400
      W(2) = 0.300
      W(3) = 0.200
      W(4) = 0.100
C                       THE FOLLOWING IS THE ROUTINE FOR
C                 CALCULATING THE DEFLECTION COEFFICIENTS FOR
C                 CHOSEN VALUES OF AXIAL LOAD AND NORMAL LOAD
      DIMENSION  A(4,4), C(4), X(4), Z(3), S(3,4)
      AX = 0.
      MG = 1
      DO 10  LL = 1,3
    4 CALL SESAME
      CALL CROUT (A, C, X, M)
      GO TO (5, 42), M
    5 ERROR = 0.
      DO 6  J = 1,4
    6 ERROR = ERROR + ABSF(X(J))
      IF (ERROR - 0.0001)  9, 9, 7
    7 DO 8  I = 1,4
    8 W(I) = W(I) + X(I)
      GO TO 4
    9 CALL CPRNT (PNORM, AX, W, ERROR, A1)
      CALL STORE (AX, W, Z, S, LL)
      AX = AX + 0.5
   10 CONTINUE
      DEL = 0.5
      KO = 1
```

```
22 GO TO (23, 25, 27), KO
23 DEL = 0.05
   DO 24  I = 1,4
   W(I) = S(2,I)
24 S(3,I) = W(I)
   AX = Z(2)
   Z(3) = AX
   AX = AX + DEL
   KO = 2
   KK = 0
   CALL EXCH (Z, S)
   GO TO 13
25 DEL = 0.01
   DO 26  I = 1,4
   W(I) = S(2,I)
26 S(3,I) = W(I)
   AX = Z(2)
   Z(3) = AX
   AX = AX + DEL
   KO = 3
   KK = 0
   CALL EXCH (Z, S)
   GO TO 13
C                    THE FOLLOWING IS THE ROUTINE WHEN IN THE
C                VICINITY OF THE BUCKLING LOAD.  W(1) IS MADE
C                THE INDEPENDENT VARIABLE IN PLACE OF THE AXIAL
C                LOAD.
27 DO 28  I = 1,4
```

```
31 ERROR = 0.
   DO 32  J = 1,4
32 ERROR = ERROR + ABSF(X(J))
   IF (ERROR - 0.0001)  36, 36, 33
33 KK = KK + 1
   IF (KK - LOOP)  34, 34, 40
34 DO 35  I = 2,4
35 W(I) = W(I) + X(I)
   AX = AX + X(1)
   GO TO 30
36 CALL CPRNT (PNORM, AX, W, ERROR, A1)
   IF (AX - Z(2))  44, 44, 37
37 Z(3) = AX
   DO 38  I = 1,4
38 S(3,I) = W(I)
   DL1 = Z(3) - Z(2)
39 CONTINUE
   GO TO 44
40 PRINT 41
41 FORMAT (40H   INFINITE LOOP OR NEEDS MORE ITERATION)
   CALL CPRNT (PNORM, AX, W, ERROR, A1)
   GO TO 44
42 PRINT 43
43 FORMAT (19H   CROUT WENT CRAZY)
44 CONTINUE
   CALL EXIT
   COMMON W, AX, B1, B2, B3, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10,
  1       A11, A12, A13, A14, A15, A16, A17, MG, A, C
```

```
      END
C
C                         SUBROUTINES
C
C                    SUBROUTINE FOR SOLVING THE LINEARIZED SYSTEM
C                    OF EQUATIONS BY THE CROUT REDOX METHOD
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE CROUT (A, C, X, M)
      DIMENSION  A(4,4), C(4), AA(4,4), CC(4), X(4)
      DO 100  I = 1,4
  100 AA(I,1) = A(I,1)
      DO 101  J = 2,4
  101 AA(1,J) = A(1,J)/A(1,1)
      DO 102  I = 2,4
      DO 102  J = 2,4
  102 AA(I,J) = 0.
      J = 2
  103 II = J
      DO 105  I = II,4
      LIM1 = J - 1
      DO 104  K = 1,LIM1
  104 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
  105 AA(I,J) = A(I,J) - AA(I,J)
      IF (AA(J,J))  106, 116, 106
  106 IF (4 - J)  110, 110, 107
  107 I = J
```

```
      J = J + 1
      JJ = J
      DO 109  J = JJ,4
      LIM2 = I - 1
      DO 108  K = 1,LIM2
  108 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
      AA(I,J) = A(I,J) - AA(I,J)
  109 AA(I,J) = AA(I,J)/AA(I,I)
      J = I + 1
      GO TO 103
  110 DO 111  I = 1,4
      X(I) = 0.
  111 CC(I) = 0.
      CC(1) = C(1)/AA(1,1)
      DO 113  I = 2,4
      LIM3 = I - 1
      DO 112  K = 1,LIM3
  112 CC(I) = CC(I) + (AA(I,K) * CC(K))
      CC(I) = C(I) - CC(I)
  113 CC(I) = CC(I)/AA(I,I)
      X(4) = CC(4)
      DO 115  I = 1,3
      II = 4 - I
      LIM4 = II + 1
      DO 114  K = LIM4,4
  114 X(II) = X(II) + (AA(II,K) * X(K))
  115 X(II) = CC(II) - X(II)
      M = 1
```

```
      GO TO 118
  116 PRINT 117
  117 FORMAT (28H   SINGULARITY - NO SOLUTION)
      M = 2
  118 RETURN
      END
C                 SUBROUTINE FOR CALCULATING THE NON-DIMENSIONAL
C                 STRAIN AND THEN PRINTING ALL THE NEEDED RESULTS
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE CPRNT (PNORM, AX, W, ERROR, A1)
      DIMENSION W(4)
      SUM = 0.
      DO 200  IJ = 1,4
      XY = 2 * IJ - 1
  200 SUM = SUM + (XY * W(IJ))**2
      STRAIN = AX + 3.1416**2 * SUM/(8. * A1)
      PRINT 201, PNORM, AX, (W(I), I=1,4), STRAIN, ERROR
  201 FORMAT (F8.2, F11.5, 4F10.3, F11.5, F12.5)
      RETURN
      END
C                 SUPROUTINE FOR STORAGE OF THREE SETS OF VALUES
C                 OF AXIAL LOAD AND THE FOUR DEFLECTION COEFFICIENTS.
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE STORE (AX, W, Z, S, LL)
```

```
      DIMENSION W(4), Z(3), S(3,4)
      Z(LL) = AX
      DO 203  I =1,4
  203 S(LL,I) = W(I)
      RETURN
      END
C                 THIS SUBROUTINE IS JUST A SHIFTING PROCEDURE
C                 OF VALUES FROM ONE STORAGE LOCATION TO ANOTHER.
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE EXCH (Z, S)
      DIMENSION Z(3), S(3,4)
      DO 204  I = 1,4
      DO 204  J = 1,2
      JA = J + 1
  204 S(J,I) = S(JA,I)
      DO 205  IJ = 1,2
      JB = IJ + 1
  205 Z(IJ) = Z(JB)
      DO 206  JI = 1,4
  206 S(3,JI) = 0.
      Z(3) = 0.
      RETURN
      END
C                 SUBROUTINE FOR COMPUTING ALL THE NECESSARY
C                 CONSTANTS IN THE MAIN PROGRAM.
*     LABEL
```

```
*     LIST8
*     FORTRAN
      SUBROUTINE CONST (U, ALPHA, PNORM, A1, A2, A3, A4, A5, A6, A7, A8,
     1A9, A10, A11, A12, A13, A14, A15, A16, A17, B1, B2, B3)
      DIMENSION B1(4), B2(4), B3(4), P(9)
      P(1) = ALPHA**2
      P(2) = 1./P(1)
      DO 300  I = 1,7
      Y1 = I**2
      I1 = I + 2
  300 P(I1) = P(1)/(P(1) + Y1)**2
      A1  = P(1)
      A2  = P(1) + P(2)
      A3  = P(1) +   81. * P(2)
      A4  = P(1) +  625. * P(2)
      A5  = P(1) + 2401. * P(2)
      A6  = 3. * P(1) +  64. * P(3)
      A7  = 3. * P(1) + 576. * P(3)
      A8  = 3. * P(1) +   9. * P(4)
      A9  = 3. * P(1) +  25. * P(4)
      A10 = 3. * P(1) +  64. * P(5)
      A11 = 4. * P(1) +    9. * P(2) +   16. * P(3) + P(4)
      A12 = 4. * P(1) +  225. * P(2) +  256. * P(3) + P(6)
      A13 = 4. * P(1) + 1225. * P(2) + 1296. * P(3) + P(8)
      A14 = 4. * P(1) +   25. * P(2) +   81. * P(4) + 16. * P(5)
      A15 = 4. * P(1) +  441. * P(2) +  625. * P(4) + 16. * P(7)
      A16 = 4. * P(1) +   49. * P(2) +  256. * P(5) + 81. * P(6)
      A17 = 6. * P(1) +  144. * P(3) +  225. * P(4) +  9. * P(6)
```

```
      DO 301  I = 1,4
      I2 = 2 * I + 1
      Y2 = 2 * I - 1
      B1(I) = 256. * P(1) * PNORM/(Y2 * 3.1416**6)
      B2(I) = 4./(3. * (1. - U**2) * P(I2))
  301 B3(I) = -(16. * Y2**2)/(3.1416**2)
      RETURN
      END
C               SUBROUTINE FOR COMPUTING THE COEFFICIENTS
C               FOR THE USE OF THE CROUT REDOX METHOD
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE SESAME
      COMMON W, AX, B1, B2, B3, A1, A2, A3, A4, A5, A6, A7, A8, A9, A10,
     1       A11, A12, A13, A14, A15, A16, A17, MG, A, C
      DIMENSION W(4), B1(4), B2(4), B3(4), A(4,4), C(4)
      GO TO (401, 400), MG
  400 A(1,1)=B3(1)*W(1)
      A(2,1)=B3(2)*W(2)
      A(3,1)=B3(3)*W(3)
      A(4,1)=B3(4)*W(4)
      GO TO 402
  401 A(1,1)=B2(1)+B3(1)*AX+3.*A2*W(1)**2-6.*A1*W(1)*W(2)+A11*W(2)**2
     1       +A14*W(3)**2+A16*W(4)**2-2.*A8*W(2)*W(3)-2.*A10*W(3)*W(4)
      A(2,1)=-3.*A1*W(1)**2+2.*A11*W(1)*W(2)-2.*A8*W(1)*W(3)
     1       +2.*A6*W(2)*W(3)-2.*A9*W(2)*W(4)+A17*W(3)*W(4)
      A(3,1)=-2.*A8*W(1)*W(2)+2.*A14*W(1)*W(3)-2.*A10*W(1)*W(4)
```

```
   1         +A6*W(2)**2+A17*W(2)*W(4)
      A(4,1)=-2.*A10*W(1)*W(3)+2.*A16*W(1)*W(4)-A9*W(2)**2+A17*W(2)*W(3)
  402 A(1,2)=-3.*A1*W(1)**2+2.*A11*W(1)*W(2)+2.*A6*W(2)*W(3)
   1         -2.*A9*W(2)*W(4)-2.*A8*W(1)*W(3)+A17*W(3)*W(4)
      A(2,2)=B2(2)+B3(2)*AX+A11*W(1)**2+3.*A3*W(2)**2+A12*W(3)**2
   1         +A15*W(4)**2+2.*A6*W(1)*W(3)-2.*A9*W(1)*W(4)
      A(3,2)=-A8*W(1)**2+2.*A6*W(1)*W(2)+2.*A12*W(2)*W(3)
   1         +A17*W(1)*W(4)+2.*A7*W(3)*W(4)
      A(4,2)=-2.*A9*W(1)*W(2)+2.*A15*W(2)*W(4)+A7*W(3)**2+A17*W(1)*W(3)
      A(1,3)=A6*W(2)**2+2.*A14*W(1)*W(3)-2.*A8*W(1)*W(2)
   1         -2.*A10*W(1)*W(4)+A17*W(2)*W(4)
      A(2,3)=-A8*W(1)**2+2.*A12*W(2)*W(3)+2.*A7*W(3)*W(4)
   1         +2.*A6*W(1)*W(2)+A17*W(1)*W(4)
      A(3,3)=B2(3)+B3(3)*AX+A14*W(1)**2+A12*W(2)**2+3.*A4*W(3)**2
   1         +A13*W(4)**2+2.*A7*W(2)*W(4)
      A(4,3)=-A10*W(1)**2+2.*A7*W(2)*W(3)+2.*A13*W(3)*W(4)+A17*W(1)*W(2)
      A(1,4)=-A9*W(2)**2+2.*A16*W(1)*W(4)-2.*A10*W(1)*W(3)+A17*W(2)*W(3)
      A(2,4)=A7*W(3)**2+2.*A15*W(2)*W(4)-2.*A9*W(1)*W(2)+A17*W(1)*W(3)
      A(3,4)=-A10*W(1)**2+2.*A13*W(3)*W(4)+A17*W(1)*W(2)+2.*A7*W(2)*W(3)
      A(4,4)=B2(4)+B3(4)*AX+A16*W(1)**2+A15*W(2)**2+A13*W(3)**2
   1         +3.*A5*W(4)**2
      C(1)=B1(1)-B2(1)*W(1)-B3(1)*W(1)*AX-A2*W(1)**3+3.*A1*W(1)**2*W(2)
   1      -A11*W(1)*W(2)**2-A6*W(2)**2*W(3)+A9*W(2)**2*W(4)
   2      -A14*W(1)*W(3)**2-A16*W(1)*W(4)**2+2.*A8*W(1)*W(2)*W(3)
   3      +2.*A10*W(1)*W(3)*W(4)-A17*W(2)*W(3)*W(4)
      C(2)=B1(2)-B2(2)*W(2)-B3(2)*AX*W(2)+A1*W(1)**3-A11*W(1)**2*W(2)
   1      +A8*W(1)**2*W(3)-A3*W(2)**3-A12*W(2)*W(3)**2-A7*W(3)**2*W(4)
   2      -A15*W(2)*W(4)**2-2.*A6*W(1)*W(2)*W(3)+2.*A9*W(1)*W(2)*W(4)
```

```
3         -A17*W(1)*W(3)*W(4)
      C(3)=B1(3)-B2(3)*W(3)-B3(3)*AX*W(3)+A8*W(1)**2*W(2)
1         -A14*W(1)**2*W(3)+A10*W(1)**2*W(4)-A6*W(1)*W(2)**2
2         -A12*W(2)**2*W(3)-A4*W(3)**3-A12*W(3)*W(4)**2
3         -A17*W(1)*W(2)*W(4)-2.*A7*W(2)*W(3)*W(4)
      C(4)=B1(4)-B2(4)*W(4)-B3(4)*AX*W(4)+A10*W(1)**2*W(3)
1         -A16*W(1)**2*W(4)+A9*W(1)*W(2)**2-A15*W(2)**2*W(4)
2         -A7*W(2)*W(3)**2-A13*W(3)**2*W(4)-A5*W(4)**3
3         -A17*W(1)*W(2)*W(3)
      RETURN
      END
```

```
*     M3888-3401,FMS,DFBUG,1,1,500,0
C
C                SOLUTION OF THE DEFLECTION EQUATION
```

$$W = W_{1,1}\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + W_{1,3}\sin\frac{\pi x}{a}\sin\frac{3\pi y}{b} + W_{3,1}\sin\frac{3\pi x}{a}\sin\frac{\pi y}{b} + W_{3,3}\sin\frac{3\pi x}{a}\sin\frac{3\pi y}{b} + W_{1,5}\sin\frac{\pi x}{a}\sin\frac{5\pi y}{b} + W_{5,1}\sin\frac{5\pi x}{a}\sin\frac{\pi y}{b}$$

```
C                THIS METHOD IS SET UP FOR A CONTINUOUS
C            CHANGE IN AXIAL LOAD FROM ZERO TO A VALUE WITHIN
C            THE VICINITY OF THE CRITICAL BUCKLING LOAD.  ZERO
C            LATERAL LOAD IS EXCLUDED.
C
C                MAIN PROGRAM
C
*      XEQ
*      LABEL
*      LIST8
*      FORTRAN
       READ 1,  IND
     1 FORMAT (I5)
       PRINT 2
     2 FORMAT (114H1  PNORM     AXIAL     W(1,1)/H    W(1,3)/H    W(3,1)/
      1H    W(3,3)/H    W(1,5)/H    W(5,1)/H    STRAIN       ERROR )
       DO 44  JOB = 1,IND
       READ 3, U, ALPHA, PNORM, LOOP, LAP
     3 FORMAT (3F8.0,2I8)
       DIMENSION W(6), R(54), B1(4), B2(5), B3(3), A(6,6), C(6), X(6)
       CALL CONST (U, ALPHA, PNORM, R, B1, B2, B3)
C              INITIAL ESTIMATES OF W(I)'S
```

```
      W(1) = 0.100
      W(2) = 0.010
      W(3) = 0.010
      W(4) = 0.001
      W(5) = 0.001
      W(6) = 0.001
C                    THE FOLLOWING IS THE ROUTINE FOR
C               CALCULATING THE DEFLECTION COEFFICIENTS FOR
C               CHOSEN VALUES OF AXIAL LOAD AND NORMAL LOAD
      DIMENSION Z(3), S(3,6)
      AX = 0.
      MG = 1
      RONE = R(1)
      DO 10  LL = 1,3
    4 CALL SESAME
      CALL CROUT (A, C, X, M)
      GO TO (5, 42), M
    5 ERROR = 0.
      DO 6  J = 1,6
    6 ERROR = ERROR + ABSF(X(J))
      IF (ERROR - 0.000001)  9, 9, 7
    7 DO 8  I = 1,6
    8 W(I) = W(I) + X(I)
      GO TO 4
    9 CALL CPRNT (PNORM, AX, W, ERROR, RONE)
      CALL STORE (AX, W, Z, S, LL)
      AX = AX + 0.5
   10 CONTINUE
```

```
   DEL = 0.5
   KO = 1
   KK = 0
11 DO 12  I = 1,6
   F1 = (S(2,I) - S(1,I))/(Z(2) - Z(1))
   F2 = (S(3,I) - S(2,I))/(Z(3) - Z(2))
   F3 = (Z(3) - Z(1))/2.
   F4 = (F2 - F1)/F3
12 W(I) = W(I) + F2 * DEL + 0.5 * F4 * DEL**2
   CALL EXCH (Z, S)
13 CALL SESAME
   CALL CROUT (A, C, X, M)
   GO TO (14, 42), M
14 ERROR = 0.
   DO 15  J = 1,6
15 ERROR = ERROR + ABSF(X(J))
   IF (ERROR - 0.000001)  19, 19, 16
16 KK = KK + 1
   IF (KK - LOOP)  17, 17, 40
17 DO 18  I = 1,6
18 W(I) = W(I) + X(I)
   GO TO 13
19 IF (W(1) - S(2,1))  22, 22, 20
20 CALL CPRNT (PNORM, AX, W, ERROR, RONE)
   Z(3) = AX
   DO 21  I = 1,6
21 S(3,I) = W(I)
   AX = AX + DEL
```

```
      KK = 0
      GO TO 11
   22 GO TO (23, 25, 27), KO
   23 DEL = 0.05
      DO 24  I = 1,6
      W(I) = S(2,I)
   24 S(3,I) = W(I)
      AX = Z(2)
      Z(3) = AX
      AX = AX + DEL
      KO = 2
      KK = 0
      CALL EXCH (Z, S)
      GO TO 13
   25 DEL = 0.01
      DO 26  I = 1,6
      W(I) = S(2,I)
   26 S(3,I) = W(I)
      AX = Z(2)
      Z(3) = AX
      AX = AX + DEL
      KO = 3
      KK = 0
      CALL EXCH (Z, S)
      GO TO 13
C                   THE FOLLOWING IS THE ROUTINE WHEN IN THE
C              VICINITY OF THE BUCKLING LOAD.  W(1) IS MADE
C              THE INDEPENDENT VARIABLE IN PLACE OF THE AXIAL
```

```
C                   LOAD.
   27 DO 28  I = 1,6
      S(3,I) = S(2,I)
      S(2,I) = S(1,I)
   28 W(I) = S(3,I)
      Z(3) = Z(2)
      Z(2) = Z(1)
      AX = Z(3)
C                   THE FOLLOWING VALUES OF DL1 AND DL2 ARE ARBITRARY.
C                   THE FIRST VALUE OF DL1 IS SO CHOSEN SUCH THAT WHEN
C                   W(1) IS DECREASED BY DL2, THE CONSEQUENT ITERATED
C                   SOLUTION WILL NOT GO BACK ON THE PREVIOUSLY COMPUTED
C                   PATH.  (NOTE - THE W(1) OR THE FIRST HARMONIC STARTS
C                   TO DECREASE IN VALUE BEFORE THE CRITICAL BUCKLING
C                   LOAD IS REACHED)
      DL1 = 0.20
      DL2 = 0.010
     .MG = 2
C                   THIS LAST DO LOOP IS HOPED TO GO BEYOND THE CRI-
C                   TICAL RANGE. START WITH LAP = 30.
      DO 39  LAST = 1,LAP
      KK = 0
      AX = AX + DL1
      W(1) = W(1) - DL2
      DO 29  J = 2,6
   29 W(I) = W(I) + (S(3,J) - S(2,J))
      CALL EXCH (Z, S)
   30 CALL SESAME
```

```
   CALL CROUT (A, C, X, M)
   GO TO (31, 42), M
31 ERROR = 0.
   DO 32  J = 1,6
32 ERROR = ERROR + ABSF(X(J))
   IF (ERROR - 0.000001)  36, 36, 33
33 KK = KK + 1
   IF (KK - LOOP)  34, 34, 40
34 DO 35  I = 2,6
35 W(I) = W(I) + X(I)
   AX = AX + X(1)
   GO TO 30
36 CALL CPRNT (PNORM, AX, W, ERROR, RONE)
   IF (AX - Z(2))  44, 44, 37
37 Z(3) = AX
   DO 38  I = 1,6
38 S(3,I) = W(I)
   DL1 = Z(3) - Z(2)
39 CONTINUE
   GO TO 44
40 PRINT 41
41 FORMAT (40H   INFINITE LOOP OR NEEDS MORE ITERATION)
   CALL CPRNT (PNORM, AX, W, ERROR, RONE)
   GO TO 44
42 PRINT 43
43 FORMAT (19H   CROUT WENT CRAZY)
44 CONTINUE
   CALL EXIT
```

```
      COMMON W, AX, B1, B2, B3, R, MG, A, C
      END
C
C                            SUBROUTINES
C
C                  SUBROUTINE FOR SOLVING THE LINEARIZED SYSTEM
C                  OF EQUATIONS BY THE CROUT REDOX METHOD
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE CROUT (A, C, X, M)
      DIMENSION  A(6,6), C(6), AA(6,6), CC(6), X(6)
      DO 100  I = 1,6
  100 AA(I,1) = A(I,1)
      DO 101  J = 2,6
  101 AA(1,J) = A(1,J)/A(1,1)
      DO 102  I = 2,6
      DO 102  J = 2,6
  102 AA(I,J) = 0.
      J = 2
  103 II = J
      DO 105  I = II,6
      LIM1 = J - 1
      DO 104  K = 1,LIM1
  104 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
  105 AA(I,J) = A(I,J) - AA(I,J)
      IF (AA(J,J))  106, 116, 106
  106 IF (6 - J)  110, 110, 107
```

```
107 I = J
    J = J + 1
    JJ = J
    DO 109  J = JJ,6
    LIM2 = I - 1
    DO 108  K = 1,LIM2
108 AA(I,J) = AA(I,J) + (AA(I,K) * AA(K,J))
    AA(I,J) = A(I,J) - AA(I,J)
109 AA(I,J) = AA(I,J)/AA(I,I)
    J = I + 1
    GO TO 103
110 DO 111  I = 1,6
    X(I) = 0.
111 CC(I) = 0.
    CC(1) = C(1)/AA(1,1)
    DO 113  I = 2,6
    LIM3 = I - 1
    DO 112  K = 1,LIM3
112 CC(I) = CC(I) + (AA(I,K) * CC(K))
    CC(I) = C(I) - CC(I)
113 CC(I) = CC(I)/AA(I,I)
    X(6) = CC(6)
    DO 115  I = 1,5
    II = 6 - I
    LIM4 = II + 1
    DO 114  K = LIM4,6
114 X(II) = X(II) + (AA(II,K) * X(K))
115 X(II) = CC(II) - X(II)
```

```
      M = 1
      GO TO 118
  116 PRINT 117
  117 FORMAT (28H   SINGULARITY - NO SOLUTION)
      M = 2
  118 RETURN
      END
C                SUBROUTINE FOR STORAGE OF THREE SETS OF VALUES
C                OF AXIAL LOAD AND THE FOUR DEFLECTION COEFFICIENTS.
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE STORE (AX, W, Z, S, LL)
      DIMENSION W(6), Z(3), S(3,6)
      Z(LL) = AX
      DO 203  I = 1,6
  203 S(LL,I) = W(I)
      RETURN
      END
C                THIS SUBROUTINE IS JUST A SHIFTING PROCEDURE
C                OF VALUES FROM ONE STORAGE LOCATION TO ANOTHER.
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE EXCH (Z, S)
      DIMENSION Z(3), S(3,6)
      DO 204  I = 1,6
      DO 204  J = 1,2
```

```
      JA = J + 1
  204 S(J,I) = S(JA,I)
      DO 205  IJ = 1,2
      JB = IJ + 1
  205 Z(IJ) = Z(JB)
      DO 206  JI = 1,6
  206 S(3,JI) = 0.
      Z(3) = 0.
      RETURN
      END
C                 SUBROUTINE FOR PRINTING
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE CPRNT (PNORM, AX, W, ERROR, RONE)
      DIMENSION W(6)
      SUM=W(1)**2+W(2)**2+W(5)**2+9.*(W(3)**2+W(4)**2)+25.*W(6)**2
      STRAIN = AX + 3.1416**2 * SUM/(8. * RONE)
      PRINT 200, PNORM, AX, (W(I), I=1,6), STRAIN, ERROR
  200 FORMAT (F8.2, F11.5, 6F12.6, F11.5, F12.7)
      RETURN
      END
C                 SUBROUTINE FOR COMPUTING ALL THE NECESSARY
C                 CONSTANTS IN THE MAIN PROGRAM
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE CONST (U, ALPHA, PNORM, R, B1, B2, B3)
```

```
      DIMENSION P(13), R(54), B1(4), B2(5), B3(3)
      P(1) = ALPHA**2
      P(2) = 1./P(1)
      DO 300  I = 3,7
      Y1 = (I - 2)**2
  300 P(I) = P(1)/(P(1) + Y1)**2
      DO 301  J = 8,11
      Y2 = (J - 6)**2
  301 P(J) = P(1)/(1. + Y2 * P(1))**2
      P(12) = P(1)/(4. * P(1) + 9.)**2
      P(13) = P(1)/(4. + 9. * P(1))**2
      R(1)  = P(1)
      R(2)  = P(2)
      R(3)  = P(3)
      R(4)  = P(4)
      R(5)  = P(8)
      R(6)  =       P(1) +      P(2)
      R(7)  =       P(1) +  81.*P(2)
      R(8)  =       P(1) + 625.*P(2)
      R(9)  =  81.*P(1) +      P(2)
      R(10) = 625.*P(1) +      P(2)
      R(11) =       P(1) +      P(8)
      R(12) =       P(1) +  81.*P(8)
      R(13) =       P(2) +      P(4)
      R(14) =       P(2) +  81.*P(4)
      R(15) =       P(1) +   3.*P(4)
      R(16) =       P(2) +   3.*P(8)
      R(17) =   3.*P(1) +  64.*P(3)
```

```
R(18) =    3.*P(2) +  64.*P(3)
R(19) =   64.*P(3) + 274.*P(4)
R(20) =   64.*P(3) + 274.*P(8)
R(21) =       P(4) +      P(8)
R(22) =       P(3) +      P(5)
R(23) =       P(3) +      P(9)
R(24) =    9.*P(1) +    64.*P(3)  +    25.*P(8)
R(25) =    9.*P(2) +    64.*P(3)  +    25.*P(4)
R(26) =   25.*P(1) +   441.*P(8)  +   256.*P(9)
R(27) =   25.*P(2) +   441.*P(4)  +   256.*P(5)
R(28) =  272.*P(3) +   625.*P(4)  +   625.*P(8)
R(29) =    9.*P(1) +    64.*P(3)  +  1225.*P(4)
R(30) =    9.*P(2) +    64.*P(3)  +  1225.*P(8)
R(31) =   25.*P(1) +    64.*P(3)  +     9.*P(10)
R(32) =   25.*P(2) +    64.*P(3)  +     9.*P(6)
R(33) =   64.*P(3) +   441.*P(4)  +   169.*P(13)
R(34) =   64.*P(3) +   441.*P(8)  +   169.*P(12)
R(35) =   25.*P(1) +   576.*P(3)  +  8281.*P(13)
R(36) =   25.*P(2) +   576.*P(3)  +  8281.*P(12)
R(37) = 1552.*P(3) + 28561.*P(12) + 28561.*P(13)
R(38) =   9.*P(1) +    4.*P(2) +   16.*P(3) +       P(8)
R(39) =   4.*P(1) +    9.*P(2) +   16.*P(3) +       P(4)
R(40) =  36.*P(1) +   36.*P(2) +  225.*P(4) +  225.*P(8)
R(41) =  25.*P(1) +    4.*P(2) +   81.*P(8) +   16.*P(9)
R(42) =   4.*P(1) +   25.*P(2) +   81.*P(4) +   16.*P(5)
R(43) =  36.*P(1) +       P(2) +  144.*P(9) +    9.*P(13)
R(44) =      P(1) +   36.*P(2) +  144.*P(5) +    9.*P(12)
R(45) =   2.*P(1) +   16.*P(3) +   49.*P(4) +    9.*P(8)
```

[illegible]

```
      R(46) =   2.*P(2) +   16.*P(3) +    9.*P(4) +   49.*P(8)
      R(47) =  16.*P(3) +   81.*P(4) +       P(8) +   81.*P(10)
      R(48) =  16.*P(3) +       P(4) +   81.*P(6) +   81.*P(8)
      R(49) =   4.*P(1) +  225.*P(2) +  256.*P(3) +       P(6)
      R(50) = 225.*P(1) +    4.*P(2) +  256.*P(3) +       P(10)
      R(51) = 256.*P(3) + 2401.*P(4) + 4096.*P(5) + 2401.*P(12)
      R(52) = 256.*P(3) + 2401.*P(8) + 4096.*P(9) + 2401.*P(13)
      R(53) =  2.*P(1' + 64.*P(3) + 256.*P(5) + 225.*P(8) + 49.*P(12)
      R(54) =  2.*P(2) + 64.*P(3) + 225.*P(4) + 256.*P(9) + 49.*P(13)
      DO 302  I = 1,3
      Y3 = 2 * I - 1
      B1(I) = 256. * P(1) * PNORM/(Y3 * 3.1416**6)
  302 B3(I) = -(16. * Y3**2)/(3.1416**2)
      B1(4) = B1(2)/3.
      DO 303  J = 1,5
      I1 = 2 * J + 1
  303 B2(J) = 4./(3. * (1. - U**2) * P(I1))
      RETURN
      END
C                 SUBROUTINE FOR COMPUTING THE COEFFICIENTS
C                 FOR THE USE OF THE CROUT REDOX METHOD
*     LABEL
*     LIST8
*     FORTRAN
      SUBROUTINE SESAME
      COMMON  W, AX, B1, B2, B3, R, MG, A, C
      DIMENSION  W(6), B1(4), B2(5), B3(3), R(54), A(6,6), C(6)
      GO TO (401, 400), MG
```

```
400 A(1,1)=B3(1)*W(1)
    A(2,1)=B3(1)*W(2)
    A(3,1)=B3(2)*W(3)
    A(4,1)=B3(2)*W(4)
    A(5,1)=B3(1)*W(5)
    A(6,1)=B3(3)*W(6)
    GO TO 402
401 A(1,1)=B2(1)+B3(1)*AX+3.*R(6)*W(1)**2-6.*R(2)*W(1)*W(2)
   1       -6.*R(1)*W(1)*W(3)+R(38)*W(2)**2+R(39)*W(3)**2
   2       +32.MR(3)*W(2)*W(3)+81.*R(21)*W(4)**2-18.*R(11)*W(2)*W(4)
   3       -18.*R(13)*W(3)*W(4)-6.*R(16)*W(2)*W(5)-6.*R(15)*W(3)*W(6)
   4       +162.*R(5)*W(4)*W(5)+162.*R(4)*W(4)*W(6)+R(41)*W(5)**2
   5       +R(42)*W(6)**2
    A(2,1)=-3.*R(2)*W(1)**2+2.*R(38)*W(1)*W(2)+32.*R(3)*W(1)*W(3)
   1       -18.*R(11)*W(1)*W(4)-R(25)*W(3)**2-2.*R(24)*W(2)*W(3)
   2       +R(40)*W(3)*W(4)-6.*R(16)*W(1)*W(5)+2.*R(18)*W(2)*W(5)
   3       +R(20)*W(3)*W(5)+R(19)*W(3)*W(6)-144.*R(23)*W(4)*W(5)
   4       -9.*R(45)*W(4)*W(6)-R(27)*W(6)**2
    A(3,1)=-3.*R(1)*W(1)**2+32.*R(3)*W(1)*W(2)+2.*R(39)*W(1)*W(3)
   1       -18.*R(13)*W(1)*W(4)-R(24)*W(2)**2-2.*R(25)*W(2)*W(3)
   2       +R(40)*W(2)*W(4)-6.*R(15)*W(1)*W(6)+R(20)*W(2)*W(5)
   3       +R(19)*W(2)*W(6)+2.*R(17)*W(3)*W(6)-9.*R(46)*W(4)*W(5)
   4       -144.*R(22)*W(4)*W(6)-R(26)*W(5)**2
    A(4,1)=-18.MR(11)*W(1)*W(2)-18.*R(13)*W(1)*W(3)
   1       +162.*R(21)*W(1)*W(4)+R(40)*W(2)*W(3)+162.*R(5)*W(1)*W(5)
   2       +162.*R(4)*W(1)*W(6)-144.*R(23)*W(2)*W(5)
   3       -9.*R(45)*W(2)*W(6)-9.*R(46)*W(3)*W(5)-144.*R(22)*W(3)*W(6)
   4       +18.*R(14)*W(4)*W(5)+18.*R(12)*W(4)*W(6)
```

```
    5        +729.*R(21)*W(5)*W(6)
      A(5,1)=-6.*R(16)*W(1)*W(2)+162.*R(5)*W(1)*W(4)+R(18)*W(2)**2
    1        +R(20)*W(2)*W(3)+9.*R(14)*W(4)**2-144.*R(23)*W(2)*W(4)
    2        -9.*R(46)*W(3)*W(4)+2.*R(41)*W(1)*W(5)-2.*R(26)*W(3)*W(5)
    3        +729.*R(21)*W(4)*W(6)
      A(6,1)=-6.*R(15)*W(1)*W(3)+162.*R(4)*W(1)*W(4)+R(17)*W(3)**2
    1        +R(19)*W(2)*W(3)+9.*R(12)*W(4)**2-9.*R(45)*W(2)*W(4)
    2        -144.*R(22)*W(3)*W(4)+2.*R(42)*W(1)*W(6)-2.*R(27)*W(2)*W(6)
    3        +729.*R(21)*W(4)*W(5)
  402 A(1,2)=-3.*R(2)*W(1)**2+2.*R(38)*W(1)*W(2)+32.*R(3)*W(1)*W(3)
    1        -18.*R(11)*W(1)*W(4)-2.*R(24)*W(2)*W(3)-R(25)*W(3)**2
    2        +R(40)*W(3)*W(4)-6.*R(16)*W(1)*W(5)+2.*R(18)*W(2)*W(5)
    3        +R(20)*W(3)*W(5)+R(19)*W(3)*W(6)-144.*R(23)*W(4)*W(5)
    4        -9.*R(45)*W(4)*W(6)-R(27)*W(6)**2
      A(2,2)=B2(4)+B3(1)*AX+R(38)*W(1)**2-2.*R(24)*W(1)*W(3)
    1        +3.*R(9)*W(2)**2+R(28)*W(3)**2-486.*R(1)*W(2)*W(4)
    2        +R(43)*W(4)**2+2.*R(18)*W(1)*W(5)-512.*R(3)*W(3)*W(5)
    3        -2.*R(29)*W(3)*W(6)+R(50)*W(5)**2+R(51)*W(6)**2
      A(3,2)=16.*R(3)*W(1)**2-2.*R(24)*W(1)*W(2)-2.*R(25)*W(1)*W(3)
    1        +R(40)*W(1)*W(4)+2.*R(28)*W(2)*W(3)+R(20)*W(1)*W(5)
    2        +R(19)*W(1)*W(6)-512.*R(3)*W(2)*W(5)-2.*R(29)*W(2)*W(6)
    3        -2.*R(30)*W(3)*W(5)-512.*R(3)*W(3)*W(6)+9.*R(54)*W(4)*W(5)
    4        +9.*R(53)*W(4)*W(6)+656.*R(3)*W(5)*W(6)
      A(4,2)=-9.*R(11)*W(1)**2+R(40)*W(1)*W(3)-243.*R(1)*W(2)**2
    1        +18.*R(43)*W(2)*W(4)-144.*R(23)*W(1)*W(5)-9.*R(31)*W(5)**2
    2        -9.*R(45)*W(1)*W(6)+9.*R(54)*W(3)*W(5)+9.*R(53)*W(3)*W(6)
    3        -9.*R(33)*W(5)*W(6)
      A(5,2)=-3.*R(16)*W(1)**2+2.*R(18)*W(1)*W(2)+R(20)*W(1)*W(3)
```

```
     1       -144.*R(23)*W(1)*W(4)-512.*R(3)*W(2)*W(3)-R(30)*W(3)**2
     2       +9.*R(54)*W(3)*W(4)+2.*R(50)*W(2)*W(5)+656.*R(3)*W(3)*W(6)
     3       -18.*R(31)*W(4)*W(5)-9.*R(33)*W(4)*W(6)-R(36)*W(6)**2
      A(6,2)=R(19)*W(1)*W(3)-9.*R(45)*W(1)*W(4)-2.*R(29)*W(2)*W(3)
     1       -256.*R(3)*W(3)**2+9.*R(53)*W(3)*W(4)-2.*R(27)*W(1)*W(6)
     2       +2.*R(51)*W(2)*W(6)+656.*R(3)*W(3)*W(5)-9.*R(33)*W(4)*W(5)
     3       -2.*R(36)*W(5)*W(6)
      A(1,3)=-3.*R(1)*W(1)**2+2.*R(39)*W(1)*W(3)+32.*R(3)*W(1)*W(2)
     1       -18.*R(13)*W(1)*W(4)-R(24)*W(2)**2-2.*R(25)*W(2)*W(3)
     2       +R(40)*W(2)*W(4)-6.*R(15)*W(1)*W(6)+2.*R(17)*W(3)*W(6)
     3       +R(20)*W(2)*W(5)+R(19)*W(2)*W(6)-9.*R(46)*W(4)*W(5)
     4       -144.*R(22)*W(4)*W(6)-R(26)*W(5)**2
      A(2,3)=16.*R(3)*W(1)**2-2.*R(25)*W(1)*W(3)-2.*R(24)*W(1)*W(2)
     1       +R(40)*W(1)*W(4)+2.*R(28)*W(2)*W(3)+R(20)*W(1)*W(5)
     2       +R(19)*W(1)*W(6)-2.*R(30)*W(3)*W(5)-512.*R(3)*W(3)*W(6)
     3       -512.*R(3)*W(2)*W(5)-2.*R(29)*W(2)*W(6)+9.*R(54)*W(4)*W(5)
     4       +9.*R(53)*W(4)*W(6)+656.*R(3)*W(5)*W(6)
      A(3,3)=B2(2)+B3(2)*AX+R(39)*W(1)**2-2.*R(25)*W(1)*W(2)
     1       +R(28)*W(2)**2+3.*R(7)*W(3)**2-486.*R(2)*W(3)*W(4)
     2       +9.*R(44)*W(4)**2+2.*R(17)*W(1)*W(6)-2.*R(30)*W(2)*W(5)
     3       -512.*R(3)*W(2)*W(6)+R(52)*W(5)**2+R(49)*W(6)**2
      A(4,3)=-9.*R(13)*W(1)**2+R(40)*W(1)*W(2)-243.*R(2)*W(3)**2
     1       +18.MR(44)*W(3)*W(4)-9.*R(46)*W(1)*W(5)-9.*R(32)*W(6)**2
     2       -144.*R(22)*W(1)*W(6)+9.*R(54)*W(2)*W(5)+9.*R(53)*W(2)*W(6)
     3       -9.*R(34)*W(5)*W(6)
      A(5,3)=R(20)*W(1)*W(2)-9.*R(46)*W(1)*W(4)-256.*R(3)*W(2)**2
     1       -2.*R(30)*W(2)*W(3)+9.*R(54)*W(2)*W(4)-2.*R(26)*W(1)*W(5)
     2       +2.*R(52)*W(3)*W(5)+656.*R(3)*W(2)*W(6)-9.*R(34)*W(4)*W(6)
```

```
     3          -2.*R(35)*W(5)*W(6)
      A(6,3)=-3.*R(15)*W(1)**2+2.*R(17)*W(1)*W(3)+R(19)*W(1)*W(2)
     1          -144.*R(22)*W(1)*W(4)-R(29)*W(2)**2-512.*R(3)*W(2)*W(3)
     2          +9.*R(53)*W(2)*W(4)+2.*R(49)*W(3)*W(6)+656.*R(3)*W(2)*W(5)
     3          -9.*R(34)*W(4)*W(5)-18.*R(32)*W(4)*W(6)-R(35)*W(5)**2
      A(1,4)=162.MR(21)*W(1)*W(4)-18.*R(11)*W(1)*W(2)
     1          -18.*R(13)*W(1)*W(3)+R(40)*W(2)*W(3)+162.*R(5)*W(1)*W(5)
     2          +162.*R(4)*W(1)*W(6)-144.*R(23)*W(2)*W(5)
     3          -9.*R(45)*W(2)*W(6)-9.*R(46)*W(3)*W(5)-144.*R(22)*W(3)*W(6)
     4          +18.*R(14)*W(4)*W(5)+18.*R(12)*W(4)*W(6)
     5          -729.*R(21)*W(5)*W(6)
      A(2,4)=-9.*R(11)*W(1)**2+R(40)*W(1)*W(3)-243.*R(1)*W(2)**2
     1          +2.*R(43)*W(2)*W(4)-144.*R(23)*W(1)*W(5)-9.*R(45)*W(1)*W(6)
     2          +9.*R(54)*W(3)*W(5)+9.*R(53)*W(3)*W(6)-9.*R(31)*W(5)**2
     3          -9.*R(33)*W(5)*W(6)
      A(3,4)=-9.*R(13)*W(1)**2+R(40)*W(1)*W(2)-243.*R(2)*W(3)**2
     1          +18.*R(44)*W(3)*W(4)-9.*R(46)*W(1)*W(5)-9.*R(34)*W(5)*W(6)
     2          -144.*R(22)*W(1)*W(6)+9.*R(54)*W(2)*W(5)+9.*R(53)*W(2)*W(6)
     3          -9.*R(32)*W(6)**2
      A(4,4)=81.*B2(1)+B3(2)*AX+81.*R(21)*W(1)**2+9.*R(43)*W(2)**2
     1          +243.*R(6)*W(4)**2+9.*R(44)*W(3)**2+18.*R(14)*W(1)*W(5)
     2          +18.MR(12)*W(1)*W(6)+81.*R(47)*W(5)**2+81.*R(48)*W(6)**2
     3          +2592.*R(3)*W(5)*W(6)
      A(5,4)=81.*R(5)*W(1)**2+18.*R(14)*W(1)*W(4)+9.*R(54)*W(2)*W(3)
     1          -144.*R(23)*W(1)*W(2)-9.*R(46)*W(1)*W(3)
     2          +729.*R(21)*W(1)*W(6)-18.*R(31)*W(2)*W(5)
     3          -9.*R(33)*W(2)*W(6)-9.*R(34)*W(3)*W(6)+162.*R(47)*W(4)*W(5)
     4          +2592.*R(3)*W(4)*W(6)
```

```
 A(6,4)=81.*R(4)*W(1)**2+18.*R(12)*W(1)*W(4)-9.*R(45)*W(1)*W(2)
1         -144.*R(22)*W(1)*W(3)+9.*R(53)*W(2)*W(3)
2         +729.*R(21)*W(1)*W(5)-9.*R(33)*W(2)*W(5)-9.*R(34)*W(3)*W(5)
3         -18.*R(32)*W(3)*W(6)+2592.*R(3)*W(4)*W(5)
4         +162.*R(48)*W(4)*W(6)
 A(1,5)=-6.*R(16)*W(1)*W(2)+162.*R(5)*W(1)*W(4)+2.*R(41)*W(1)*W(5)
1         +R(18)*W(2)**2+R(20)*W(2)*W(3)-144.*R(23)*W(2)*W(4)
2         -9.*R(46)*W(3)*W(4)+9.*R(14)*W(4)**2-2.*R(26)*W(3)*W(5)
3         +729.*R(21)*W(4)*W(6)
 A(2,5)=-3.*R(16)*W(1)**2+2.*R(18)*W(1)*W(2)+R(20)*W(1)*W(3)
1         -144.*R(23)*W(1)*W(4)-R(30)*W(3)**2-512.*R(3)*W(2)*W(3)
2         +9.*R(54)*W(3)*W(4)+2.*R(50)*W(2)*W(5)+656.*R(3)*W(3)*W(6)
3         -18.*R(31)*W(4)*W(5)-9.*R(33)*W(4)*W(6)-R(36)*W(6)**2
 A(3,5)=R(20)*W(1)*W(2)-9.*R(46)*W(1)*W(4)-2.*R(26)*W(1)*W(5)
1         -256.*R(3)*W(2)**2-2.*R(30)*W(2)*W(3)+9.*R(54)*W(2)*W(4)
2         +2.*R(52)*W(3)*W(5)+656.*R(3)*W(2)*W(6)-9.*R(34)*W(4)*W(6)
3         -2.*R(35)*W(5)*W(6)
 A(4,5)=81.*R(5)*W(1)**2-144.*R(23)*W(1)*W(2)-9.*R(46)*W(1)*W(3)
1         +18.*R(14)*W(1)*W(4)+729.*R(21)*W(1)*W(6)
2         +9.*R(54)*W(2)*W(3)-18.*R(31)*W(2)*W(5)-9.*R(33)*W(2)*W(6)
3         -9.*R(34)*W(3)*W(6)+162.*R(47)*W(4)*W(5)
4         +2592.*R(3)*W(4)*W(6)
 A(5,5)=B2(5)+B3(1)*AX+R(41)*W(1)**2-2.*R(26)*W(1)*W(3)
1         +R(50)*W(2)**2+R(52)*W(3)**2-18.*R(31)*W(2)*W(4)
2         +81.*R(47)*W(4)**2-2.*R(35)*W(3)*W(6)+3.*R(10)*W(5)**2
3         +R(37)*W(6)**2
 A(6,5)=729.*R(21)*W(1)*W(4)+656.*R(3)*W(2)*W(3)
1         -9.*R(33)*W(2)*W(4)-9.*R(34)*W(3)*W(4)+1296.*R(3)*W(4)**2
```

```
2        -2.*R(35)*W(3)*W(5)-2.*R(36)*W(2)*W(6)+2.*R(37)*W(5)*W(5)
 A(1,6)=-6.*R(15)*W(1)*W(3)+162.*R(4)*W(1)*W(4)+2.*R(42)*W(1)*W(6)
1        +R(17)*W(3)**2+R(19)*W(2)*W(3)-9.*R(45)*W(2)*W(4)
2        -144.*R(22)*W(3)*W(4)+9.*R(12)*W(4)**2-2.*R(27)*W(2)*W(6)
3        +729.*R(21)*W(4)*W(5)
 A(2,6)=R(19)*W(1)*W(3)-9.*R(45)*W(1)*W(4)-2.*R(27)*W(1)*W(6)
1        -256.*R(3)*W(3)**2-2.*R(29)*W(2)*W(3)+9.*R(53)*W(3)*W(4)
2        +2.*R(51)*W(2)*W(6)+656.*R(3)*W(3)*W(5)-9.*R(33)*W(4)*W(5)
3        -2.*R(36)*W(5)*W(6)
 A(3,6)=-3.*R(15)*W(1)**2+R(19)*W(1)*W(2)+2.*R(17)*W(1)*W(3)
1        -144.*R(22)*W(1)*W(4)-R(29)*W(2)**2-512.*R(3)*W(2)*W(3)
2        +9.*R(53)*W(2)*W(4)+2.*R(49)*W(3)*W(6)+656.*R(3)*W(2)*W(5)
3        -18.*R(32)*W(4)*W(6)-9.*R(34)*W(4)*W(5)-R(35)*W(5)**2
 A(4,6)=81.*R(4)*W(1)**2-9.*R(45)*W(1)*W(2)-144.*R(22)*W(1)*W(3)
1        +18.*R(12)*W(1)*W(4)+729.*R(21)*W(1)*W(5)
2        +9.*R(53)*W(2)*W(3)-18.*R(32)*W(3)*W(6)-9.*R(33)*W(2)*W(5)
3        -9.*R(34)*W(3)*W(5)+162.*R(48)*W(4)*W(6)
4        +2592.*R(3)*W(4)*W(5)
 A(5,6)=729.*R(21)*W(1)*W(4)+656.*R(3)*W(2)*W(3)
1        -9.*R(33)*W(2)*W(4)-9.*R(34)*W(3)*W(4)+1296.*R(3)*W(4)**2
2        -2.*R(36)*W(2)*W(6)-2.*R(35)*W(3)*W(5)+2.*R(37)*W(5)*W(6)
 A(6,6)=B2(3)+B3(3)*AX+R(42)*W(1)**2-2.*R(27)*W(1)*W(2)
1        +R(51)*W(2)**2+R(49)*W(3)**2-18.*R(32)*W(3)*W(4)
2        +81.*R(48)*W(4)**2-2.*R(36)*W(2)*W(5)+R(37)*W(5)**2
3        +3.*R(8)*W(6)**2
 D1=B1(1)-B2(1)*W(1)-B3(1)*AX*W(1)-R(6)*W(1)**3
1    +3.*R(2)*W(1)**2*W(2)+3.*R(1)*W(1)**2*W(3)-R(38)*W(1)*W(2)**2
2    -R(39)*W(1)*W(3)**2-32.*R(3)*W(1)*W(2)*W(3)
```

```
3    -81.*R(21)*W(1)*W(4)**2+18.*R(11)*W(1)*W(2)*W(4)
4    +18.*R(13)*W(1)*W(3)*W(4)+R(24)*W(2)**2*W(3)
5    +R(25)*W(2)*W(3)**2-R(40)*W(2)*W(3)*W(4)
6    +6.*R(16)*W(1)*W(2)*W(5)+6.*R(15)*W(1)*W(3)*W(6)
7    -162.*R(5)*W(1)*W(4)*W(5)-162.*R(4)*W(1)*W(4)*W(6)
 E1=-R(41)*W(1)*W(5)**2-R(42)*W(1)*W(6)**2-R(18)*W(2)**2*W(5)
1    -R(17)*W(3)**2*W(6)-R(20)*W(2)*W(3)*W(5)
2    -R(19)*W(2)*W(3)*W(6)+144.*R(23)*W(2)*W(4)*W(5)
3    +9.*R(45)*W(2)*W(4)*W(6)+9.*R(46)*W(3)*W(4)*W(5)
4    +144.*R(22)*W(3)*W(4)*W(6)-9.*R(14)*W(4)**2*W(5)
5    -9.*R(12)*W(4)**2*W(6)+R(27)*W(2)*W(6)**2
6    +R(26)*W(3)*W(5)**2-729.*R(21)*W(4)*W(5)*W(6)
 D2=B1(2)-B2(4)*W(2)-B3(1)*AX*W(2)+R(2)*W(1)**3
1    -R(38)*W(1)**2*W(2)-16.*R(3)*W(1)**2*W(3)
2    +9.*R(11D*W(1)**2*W(4)+R(25)*W(1)*W(3)**2
3    +2.*R(24)*W(1)*W(2)*W(3)-R(40)*W(1)*W(3)*W(4)
4    -R(9)*W(2)**3-R(28)*W(2)*W(3)**2-R(43)*W(2)*W(4)**2
5    +243.*R(1)*W(2)**2*W(4)+3.*R(16)*W(1)**2*W(5)
6    -2.*R(18D*W(1)*W(2)*W(5)-R(20)*W(1)*W(3)*W(5)
7    -R(19)*W(1)*W(3)*W(6)+144.*R(23)*W(1)*W(4)*W(5)
 E2=9.*R(45)*W(1)*W(4)*W(6)+R(27)*W(1)*W(6)**2
1    +R(30)*W(3)**2*W(5)+256.*R(3)*W(3)**2*W(6)
2    +512.*R(3)*W(2)*W(3)*W(5)+2.*R(29)*W(2)*W(3)*W(6)
3    -9.*R(54)*W(3)*W(4)*W(5)-9.*R(53)*W(3)*W(4)*W(6)
4    -R(50)*W(2)*W(5)**2-R(51)*W(2)*W(6)**2
5    -656.*R(3)*W(3)*W(5)*W(6)+9.*R(31)*W(4)*W(5)**2
6    +9.*R(33)*W(4)*W(5)*W(6)+R(36)*W(5)*W(6)**2
 D3=B1(2)-B2(2)*W(3)-B3(2)*AX*W(3)+R(1)*W(1)**3
```

```
     1    -16.*R(3)*W(1)**2*W(2)-R(39)*W(1)**2*W(3)
     2    +9.*R(13)*W(1)**2*W(4)+R(24)*W(1)*W(2)**2
     3    +2.*R(25)*W(1)*W(2)*W(3)-R(40)*W(1)*W(2)*W(4)
     4    -R(28)*W(2)**2*W(3)-R(7)*W(3)**3+243.*R(2)*W(3)**2*W(4)
     5    -9.*R(44)*W(3)*W(4)**2+3.*R(15)*W(1)**2*W(6)
     6    -R(20)*W(1)*W(2)*W(5)-R(19)*W(1)*W(2)*W(6)
     7    -2.*R(17)*W(1)*W(3)*W(6)+9.*R(46)*W(1)*W(4)*W(5)
      E3=144.*R(22)*W(1)*W(4)*W(6)+R(26)*W(1)*W(5)**2
     1    +256.*R(3)*W(2)**2*W(5)+R(29)*W(2)**2*W(6)
     2    +2.*R(30)*W(2)*W(3)*W(5)+512.*R(3)*W(2)*W(3)*W(6)
     3    -9.*R(54)*W(2)*W(4)*W(5)-9.*R(53)*W(2)*W(4)*W(6)
     4    -R(52)*W(3)*W(5)**2-R(49)*W(3)*W(6)**2
     5    -656.*R(3)*W(2)*W(5)*W(6)+9.*R(32)*W(4)*W(6)**2
     6    +9.*R(34)*W(4)*W(5)*W(6)+R(35)*W(5)**2*W(6)
      D4=B1(4)-81.*B2(1)*W(4)-B3(2)*AX*W(4)+9.*R(11)*W(1)**2*W(2)
     1    +9.*R(13)*W(1)**2*W(3)-81.*R(21)*W(1)**2*W(4)
     2    -R(40)*W(1)*W(2)*W(3)+81.*R(1)*W(2)**3+81.*R(2)*W(3)**3
     3    -9.*R(43)*W(2)**2*W(4)-81.*R(6)*W(4)**3-9.*R(44)*W(3)**2*W(4)
     4    -81.*R(5)*W(1)**2*W(5)-81.*R(4)*W(1)**2*W(6)
     5    +144.*R(23)*W(1)*W(2)*W(5)+9.*R(45)*W(1)*W(2)*W(6)
     6    +9.*R(46)*W(1)*W(3)*W(5)+144.*R(22)*W(1)*W(3)*W(6)
      E4=-18.*R(14)*W(1)*W(4)*W(5)-18.*R(12)*W(1)*W(4)*W(6)
     1    -729.*R(21)*W(1)*W(5)*W(6)-9.*R(54)*W(2)*W(3)*W(5)
     2    -9.*R(53)*W(2)*W(3)*W(6)+9.*R(31)*W(2)*W(5)**2
     3    +9.*R(32)*W(3)*W(6)**2+9.*R(33)*W(2)*W(5)*W(6)
     4    +9.*R(34)*W(3)*W(5)*W(6)-81.*R(47)*W(4)*W(5)**2
     5    -81.*R(48)*W(4)*W(6)**2-2592.*R(3)*W(4)*W(5)*W(6)
      D5=B1(3)-B2(5)*W(5)-B3(1)*AX*W(5)+3.*R(16)*W(1)**2*W(2)
```

```
     1   -81.*R(5)*W(1)**2*W(4)-R(18)*W(1)*W(2)**2-R(20)*W(1)*W(2)*W(3)
     2   -9.*R(14)*W(1)*W(4)**2+144.*R(23)*W(1)*W(2)*W(4)
     3   +9.*R(46)*W(1)*W(3)*W(4)+256.*R(3)*W(2)**2*W(3)
     4   +R(30)*W(2)*W(3)**2-9.*R(54)*W(2)*W(3)*W(4)
     5   -R(41)*W(1)**2*W(5)+2.*R(26)*W(1)*W(3)*W(5)
     6   -729.*R(21)*W(1)*W(4)*W(6)-R(50)*W(2)**2*W(5)
      E5=-R(52)*W(3)**2*W(5)-656.*R(3)*W(2)*W(3)*W(6)
     1   +18.*R(31)*W(2)*W(4)*W(5)+9.*R(33)*W(2)*W(4)*W(6)
     2   +9.*R(34)*W(3)*W(4)*W(6)-81.*R(47)*W(4)**2*W(5)
     3   -1296.*R(3)*W(4)**2*W(6)+R(36)*W(2)*W(6)**2
     4   +2.*R(35)*W(3)*W(5)*W(6)-R(10)*W(5)**3-R(37)*W(5)*W(6)**2
      D6=B1(3)-B2(3)*W(6)-B3(3)*AX*W(6)+3.*R(15)*W(1)**2*W(3)
     1   -81.*R(4)*W(1)**2*W(4)-R(17)*W(1)*W(3)**2-R(19)*W(1)*W(2)*W(3)
     2   -9.*R(12)*W(1)*W(4)**2+9.*R(45)*W(1)*W(2)*W(4)
     3   +144.*R(22)*W(1)*W(3)*W(4)+R(29)*W(2)**2*W(3)
     4   +256.*R(3)*W(2)*W(3)**2-9.*R(53)*W(2)*W(3)*W(4)
     5   -R(42)*W(1)**2*W(6)+2.*R(27)*W(1)*W(2)*W(6)
     6   -729.*R(21)*W(1)*W(4)*W(5)-R(51)*W(2)**2*W(6)
      E6=-R(49)*W(3)**2*W(6)-656.*R(3)*W(2)*W(3)*W(5)
     1   +9.*R(33)*W(2)*W(4)*W(5)+9.*R(34)*W(3)*W(4)*W(5)
     2   +18.*R(32)*W(3)*W(4)*W(6)-1296.*R(3)*W(4)**2*W(5)
     3   -81.*R(48)*W(4)**2*W(6)+R(35)*W(3)*W(5)**2
     4   +2.*R(36)*W(2)*W(5)*W(6)-R(37)*W(5)**2*W(6)-R(8)*W(6)**3
      C(1) = D1 + E1
      C(2) = D2 + E2
      C(3) = D3 + E3
      C(4) = D4 + E4
      C(5) = D5 + E5
```

```
C(6) = D6 + E6
RETURN
END
DATA
```

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